Communication circuits

Chapter 4

EE312

by

A. DAHIMENE

Institut de Génie Electrique et Electronique

Université M'hamed Bougara
Boumerdes

Chapter 4 3

Chapter 4

Sinusoidal Oscillators

Sinewave generators are commonly used in communication circuits. We find them in both transmitters and receivers. There exist many types of oscillators: multivibrators, relaxation oscillators, etc. Most of them generate waveforms that are not sinusoidal. In this chapter, we are going to analyze only the ones that can generate sinewaves.

4.1 Basic definitions

A sinusoidal oscillator is a one port device that produces a sinewave. The output signal is then:

$$x(t) = A\cos\left(\omega_0 t + \theta\right)$$

It is a simple matter to show that the above signal satisfies the differential equation $x''(t) + \omega_0^2 x(t) = 0$. This equation represents a harmonic oscillator such as the small amplitude pendulum in vacuum, a mass and spring without friction, etc. In our case, we are interested in electrical (RC and LC) oscillators and also in electromechanical ones (crystal).

The oscillators that we will study are usually composed of a passive network that is used to set the frequency of oscillation and an amplifier that is used to compensate for the losses in the passive network.

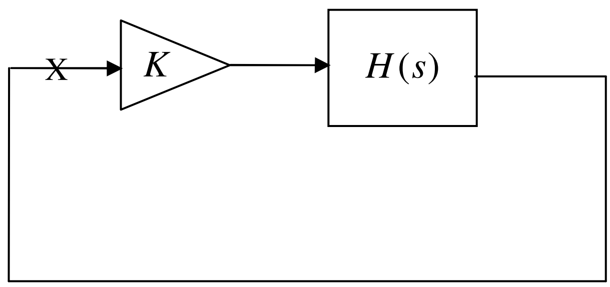


Fig. 4- 1 Typical sinusoidal oscillator

The amplifier is assumed to be memoriless. Any memory element in the real amplifier will be integrated inside the passive network. The above network can be analyzed using linear techniques. However, such analysis would correspond only at the conditions that set the start of oscillations. When the amplitude of the sinewave is small, we have seen that all of the active devices can be considered as linear (incrementally linear). However, as the amplitude increases, this assumption is no longer true. We then have to resort to nonlinear analysis.

A general nonlinear analysis of oscillators would lead us to the study of nonlinear differential equations. This is an extremely complex method of analysis. In general, we use much simpler techniques.

A general study of a sinusoidal oscillator starts in general by a small signal analysis (linear). This step allows us to compute the frequency of oscillation and also the conditions on the gain of the amplifier in order for the oscillations to start.

The second step takes into consideration the nonlinearities and studies conditions for setting the amplitude of the sinewave at a predetermined level. We can also try to determine the distortion of the waveform at this point of analysis.

4.2 Linear feedback analysis

The block diagram shown in Fig. 4- 1 suggests the following closed loop:

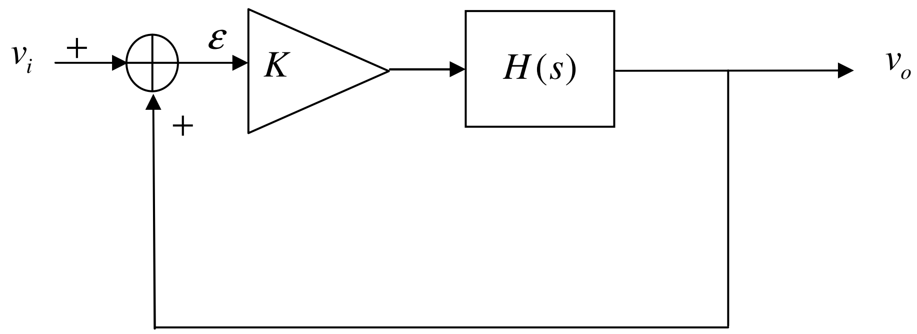


Fig. 4- 2 Closed loop

The block diagram shown in Fig. 4- 2 is equivalent to the one of Fig. 4- 1 if the input signal v_i is zero. Using results from linear feedback theory, we can define an open loop gain:

$$A_L(s) = KH(s) \quad (1)$$

and a closed loop gain:

$$G(s) = \frac{A_L(s)}{1 - A_L(s)} = \frac{KH(s)}{1 - KH(s)} \quad (2)$$

In our analysis, we are going to consider only passive networks that can be built using lumped elements. This means that the methods of analysis cannot be used for studying oscillators implemented using transmission lines or delays, etc. In the case of lumped elements, the transfer function of the passive network is a ratio of polynomials.

$$H(s) = \frac{N(s)}{D(s)} \quad (3)$$

where $N(s)$ and $D(s)$ are polynomials. It is clear that the roots of $N(s)$ are the zeros of the open loop gain $A_L(s)$ and the roots of $D(s)$ are the poles of the open loop gain $A_L(s)$. Replacing (3) in (2) provides:

$$G(s) = \frac{KN(s)}{D(s) - KN(s)}$$

The above equation can be written as:

$$\left[D(s) - KN(s)\right] v_0(s) = KN(s) v_i(s) \tag{4}$$

Assimilating the operator s to the differentiation operator, we can see from (4) that we can have an output v_o different from zero when v_i is zero if the following constant coefficient differential equation is satisfied:

$$\left[D(s) - KN(s)\right] v_0(s) = 0 \tag{5}$$

If the roots of $D(s) - KN(s)$ are distinct, the general solution of the differential equation (5) is:

$$v_0(t) = \sum_{k=1}^{n} C_k \exp\left(c_k t\right) u(t) \tag{6}$$

where c_k are the roots of (5). They are the closed loop gain poles and C_k are coefficients that depend on the initial conditions.

Since the passive network is built using real components (R, L, C and M), its transfer function is a ratio of polynomial with real coefficients. If we want to have as a solution a sinewave, we must have two closed loop poles on the $j\omega$ axis at $\pm j\omega_0$ and we have to make sure that the other closed loop poles will not appear in the signal $v_o(t)$. This can theoretically be achieved by setting the initial coefficients such that $C_k = 0$ except for the selected pair of poles. However, this solution is not realistic. A much more sensible solution is to make sure that all the unwanted poles are "stable", i.e. with a negative real part. So, if we wait a while, all the unwanted signals will decay and only the required signal will remain. From the above discussion, we observe that the position of the closed loop poles is fundamental. These closed loop poles are the solutions of the equation:

$$D(s) - KN(s) = 0 \tag{7}$$

This equation depends on the gain K. So, the position of the closed loop poles will also depend on the gain K. their locus in the s-plane is called the "*root locus*". There are some general rules for drawing a root locus. You will learn them in control courses. However, we can state some of them.

When $K = 0$, equation (7) becomes $D(s) = 0$. This means that the closed loop poles are on the open loop poles for small gains. When K becomes very large, equation (7) is practically $N(s) = 0$. So, the closed loop poles will be on the open loop zeroes. For some value of the gain, the root locus will cross the $j\omega$ axis at the frequency ω_0.

There are some minimal requirements on the transfer function $H(s)$ so that there can be oscillations and therefore closed loop poles on the $j\omega$ axis. It is shown that the transfer function must have at least two poles and one zero. Consider the following example:

$$H(s) = \frac{s^2 + cs + d}{s^2 + as + b}$$

Equation (7) becomes:

$$(1-K)s^2 + (a-Kc)s + b - Kd = 0$$

which corresponds to the differential equation:

$$(1-K)v_0'' + (a-Kc)v_0' + (b-Kd)v_0 = 0$$

In order to have oscillations, we must have:

$a - Kc = 0$ and $\dfrac{b-Kd}{1-K} > 0$. If these two conditions are satisfied, the equation becomes:

$$s^2 + \omega_0^2 = 0 \text{ where } \omega_0 = \sqrt{\frac{b-Kd}{1-K}}$$

If we have one pair of closed loop poles on the $j\omega$ axis, this implies that $1 - A_L(s) = 0$ for $s = j\omega_0$. In other words:

$$A_L(j\omega_0) = 1 \tag{8}$$

The above relation can be expressed as:

$$|A_L(j\omega_0)| = 1 \text{ and } \arg[A_L(j\omega_0)] = 0 \mod 2\pi \tag{9}$$

Equations (8) and (9) are necessary conditions. They are known as the "*Barkhausen Conditions*". Equation (9) simply means that, if we look at the diagram of Fig. 4- 1 and if we consider signals on the right and on the left of the point X, they are identical. There exists a sinewave which travels around the loop with no attenuation and no phase shift. Of course, in order to have a good sinewave oscillator, this condition should exist only for one single frequency. Sometimes, it is easier to use the real and imaginary part of the open loop transfer function.

Example:

Consider the open loop gain:

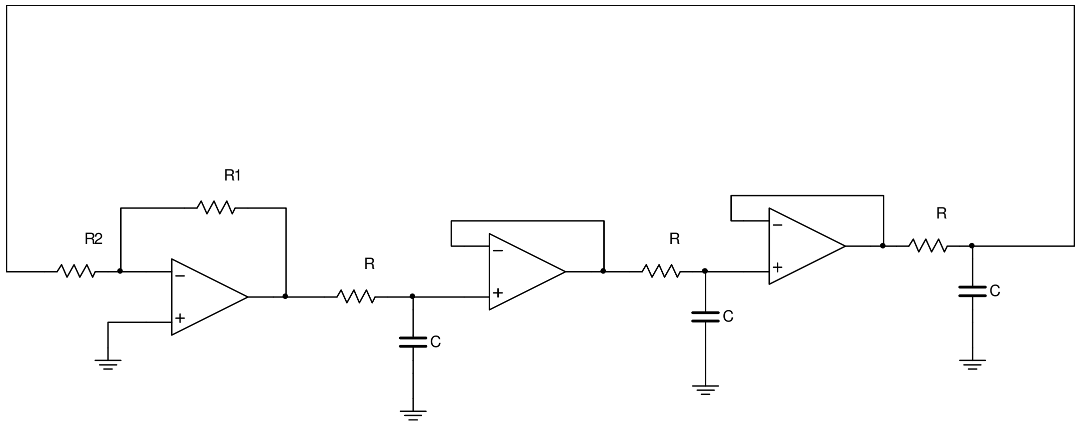
$$A_L(s) = \frac{K}{(1+RCs)^3}$$

$$A_L(j\omega)=\frac{K}{(1+jRC\omega)^2}=\frac{K\left(1-3\omega^2(RC)^2\right)}{\left(1+(RC\omega)^2\right)^3}-j\frac{KRC\omega\left(3-(RC\omega)^2\right)}{\left(1+(RC\omega)^2\right)^3}$$

The application of the Barkhausen conditions provides:

$\mathrm{Im}[A_L(\omega_0)]=0$ gives $\omega_0=\frac{\sqrt{3}}{RC}$ and $\mathrm{Re}[A_L(j\omega_0)]=1$ gives $K=-8$

This circuit can be realized by cascading three first order lowpass circuits separated by buffers.



The gain K is given by the ration of the two resistances $R1$ and $R2$.

$$K=-\frac{R1}{R2}$$

In order to neglect the load on the last RC stage, we assume that the resistance $R2$ is very large. If it is impossible to achieve this, we must add a high input impedance buffer before.

The previous example shows that the value of the gain K is critical. If its absolute value is smaller than 8, the closed loop poles will be in the left side of the s-plane and the oscillation will not start. If the gain is too large, they will be in the right half, the sinewave envelop will be a growing exponential and the amplitude will increase until it will be limited by the amplifiers nonlinearities. So, we need an infinite precision in order to set the conditions of oscillation.

Even if we are able to set exactly the gain, the linear theory cannot predict the amplitude of the sinewave. Theoretically, this amplitude is set by the initial conditions. These conditions cannot be predicted for general circuits. So, this linear analysis is used only to determine the frequency of oscillations along with conditions for starting these oscillations.

4.3 General conditions for oscillator design

There are certain conditions that must be satisfied in order to design a practical oscillator. First, the oscillator must always start when we turn the power on. Next, the amplitude of the waveform should be under our control and not set by random initial conditions. We must also have good control on the distortion and finally, the frequency of oscillation should not depend on parasitic elements and on environmental conditions.

In general, the analysis of an oscillator should be performed in two steps.

- We do a linear small signal analysis and we must set the selected closed loop poles on the right half of the complex plane (not to far from the $j\omega$ axis). This will ensure that the oscillator will start what ever initial conditions. Even if the memory elements (capacitances, inductances, etc.) have zero initial conditions, the existence of thermal noise will ensure that the oscillation will start. The fact that the closed loop poles are on the right side implies that the waveform will have an exponentially growing envelop.

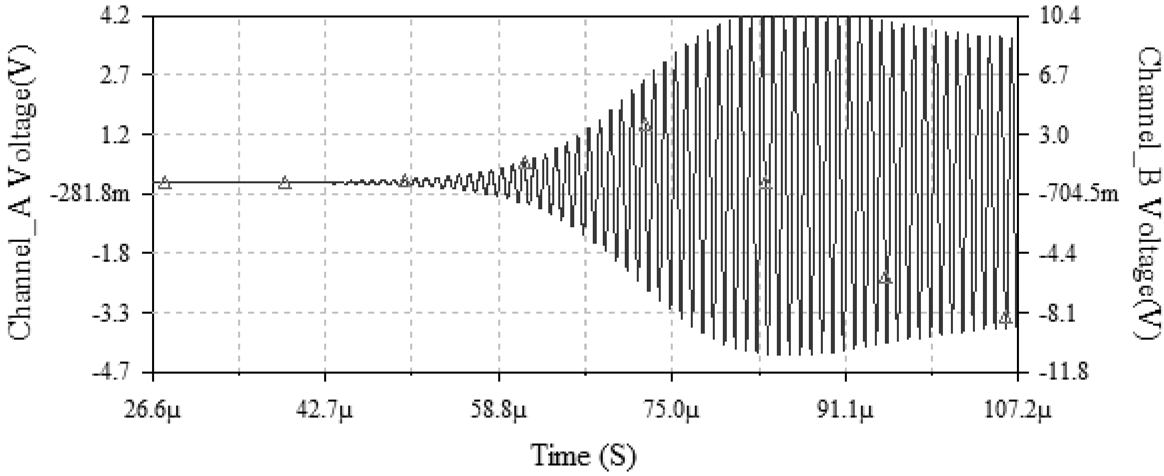


Fig. 4- 3 LC oscillator starting

Fig. 4- 3 shows the initial time of an LC oscillator (the one used in lab #4). It takes practically 100 µs for the amplitude to stabilize at its final value.

- There should exist a mechanism of amplitude control that will fix the amplitude at some pre-selected value. This mechanism can be an automatic gain control or we can use the nonlinearities of the amplifying device to push the closed loop poles back on the $j\omega$ axis.

An instructive example is the Wien bridge oscillator of lab #3.

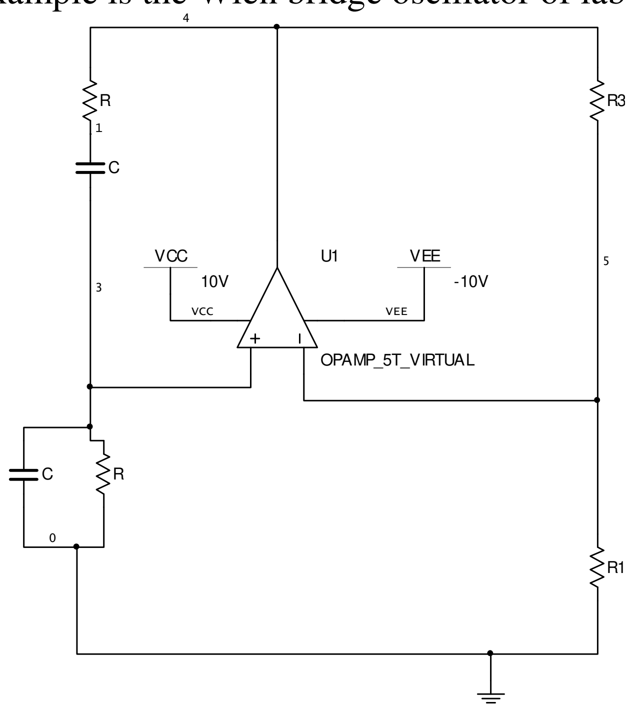


Fig. 4- 4 Wien Bridge Oscillator

It is shown in lab #3 that the open loop transfer function of the Wien bridge oscillator is:

$$A_L(s) = \frac{-A}{3+\delta}\frac{s^2 - s\delta\omega_0 + \omega_0^2}{s^2 + 3s\omega_0 + \omega_0^2}$$

Where $\omega_0 = \dfrac{1}{RC}$ and $R3 = (2+\delta)R1$. The previous method of analysis (Barkhausen conditions) gives that the circuit will oscillate at ω_0 if the gain of the amplifier is $A_{\min} = \dfrac{9}{\delta} + 3$. If the gain is higher than $A_{\min}$, the closed loop poles will be in the right half of the s-plane.

If we use an op-amp, its differential gain is very high. So, $A >> A_{\min}$. In fact, from what we have learned earlier, when the gain is very large, the closed loop poles will be practically on top of the open loop zeroes. These zeroes are:

$$z_{1,2} = \frac{\delta}{2}\omega_0 \pm j\frac{\sqrt{4-\delta^2}}{2}\omega_0$$

So, if we want the oscillator to start, we must put the closed loop poles on the right half of the s-plane. This implies that the open loop zeroes must also have a real part that is positive. We must have $\delta > 0$ or $R3 > 2R1$. However, the value of δ must not be too large. If $\delta > 2$, the open loop zeroes will be real and the oscillations will not start. In the lab experiment, you will design an oscillator with $\delta = 1$. In this case, we will have exponentially growing oscillations at the frequency of $\frac{\sqrt{3}}{2}\omega_0$. This waveform will be limited by the saturation of the op-amp and the waveform displayed by the oscilloscope will be:

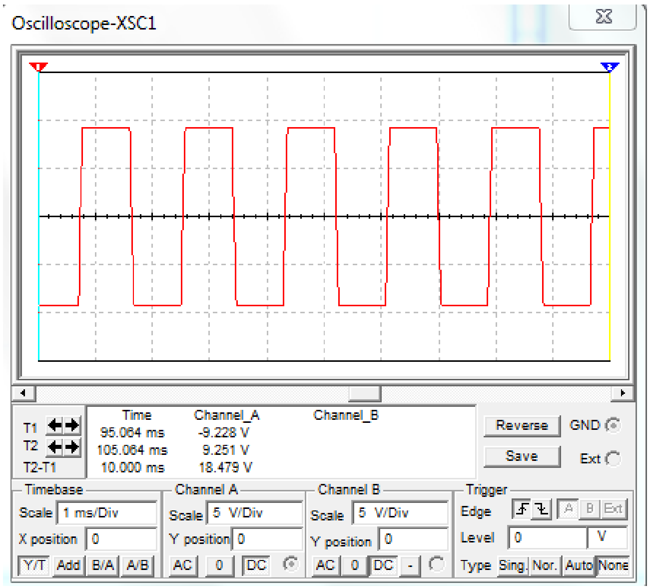


Fig. 4- 5 Output waveform for $\delta = 1$

A better signal will be obtain in the second part of lab #3 by the use of a voltage controlled resistance in place of $R1$. When the system is operating correctly, the value of δ will be very close to zero and the sinewave will be very pure.

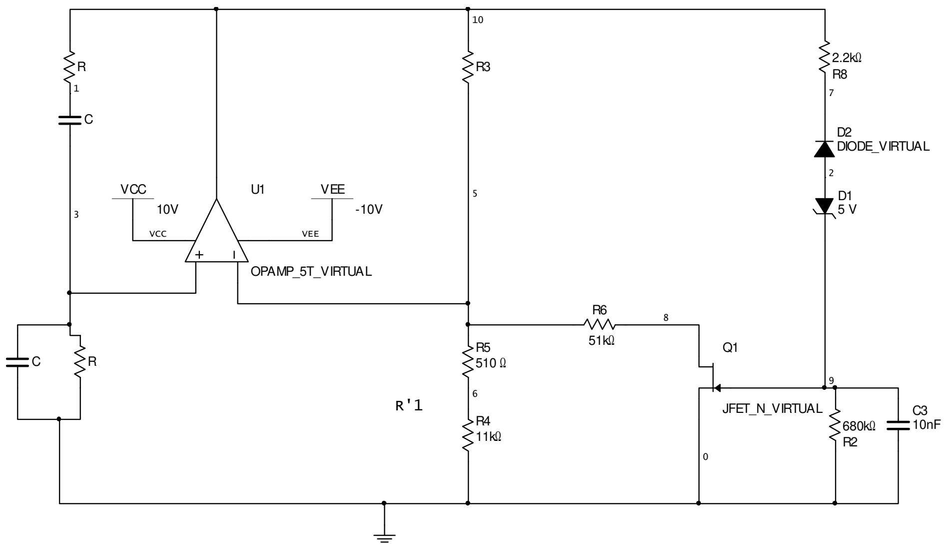


Fig. 4- 6 AGC using FET

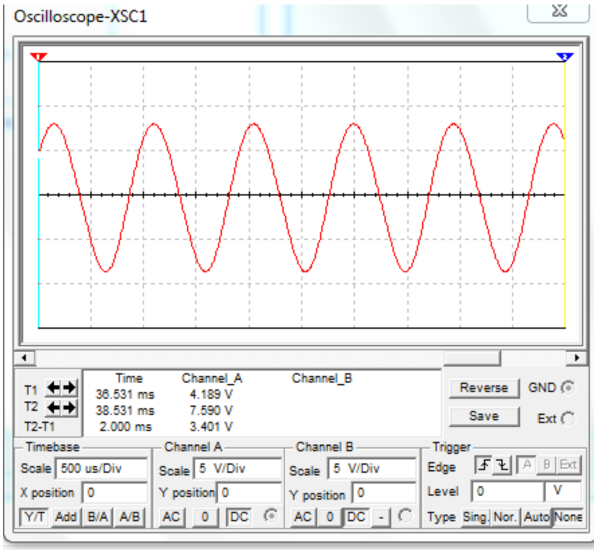


Fig. 4- 7 Output waveform

4.4 LC oscillators

LC oscillators use a parallel RLC circuit along with a transformer as passive network. The amplifier is one of the nonlinear controlled sources studied in chapter 3. So, the basic circuit is shown below (Fig. 4- 8). G_{in} represents the load presented by the input of the active device at the oscillating frequency.

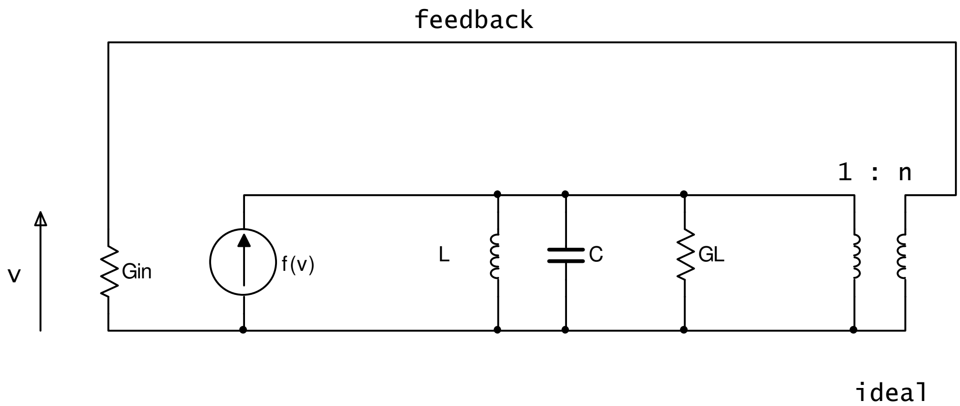


Fig. 4- 8 Typical LC oscillator

If we analyze the response of the passive network, we remark that the parallel RLC circuit has a zero phase response at the resonant frequency and at any other frequency it is either capacitive or inductive. So, from the phase condition of the Barkhausen relations, the frequency of oscillation will be the resonant frequency:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

If the total Q of the tank circuit is larger or equal to ten, the voltage at the input of the active device will be sinusoidal, even if the output current is not. This means that the only component of the current that will produce a voltage across the tank circuit is the fundamental. For all voltage controlled current sources, this current is given by:

$$I_1 \cos\omega_0 t = G_m V_1 \cos\omega_0 t$$

G_m is the large signal transconductance and $V_1\cos\omega_0 t$ is the input voltage across the input of the active device. This means that the voltage across the tank circuit is:

$$v_L(t) = \frac{I_1}{G_L + n^2 G_{in}} \cos\omega_0 t = \frac{G_m V_1}{G_L + n^2 G_{in}} \cos\omega_0 t$$

At the output of the ideal transformer, we obtain the input voltage $V_1\cos\omega_0 t$, and this voltage is equal to $nv_L(t)$. So, we obtain the equation:

$$\frac{nG_m}{G_L + n^2 G_{in}} = 1 \text{ or } G_m = \frac{G_L}{n} + nG_{in}$$

If we divide the above relation by the small signal transconductance, we obtain:

$$\frac{G_m}{g_m} = \frac{G_L}{ng_m} + \frac{nG_{in}}{g_m} \tag{10}$$

Equation (10) must be satisfied for some value of the input. We have seen that the ratio G_m/g_m depends on the amplitude V_1 and that it decreases as the amplitude increases. Furthermore, for very small amplitude, this ratio is equal to one. This means that the small signal transconductance must satisfy $g_{in} \geq \frac{G_L}{n} + nG_{in}$ for the oscillator to start.

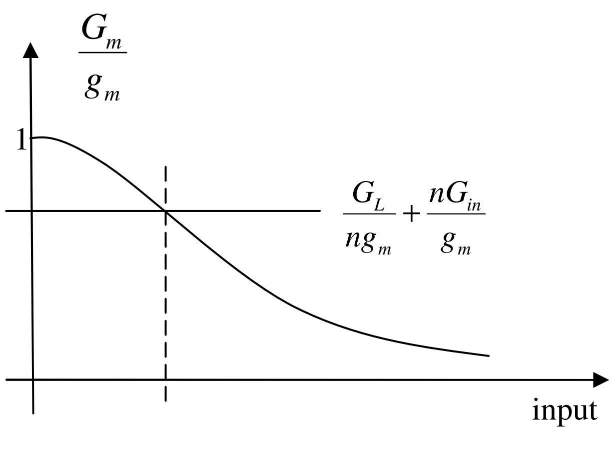


Fig. 4- 9 Typical large signal transconductance

Fig. 4- 9 shows a typical ratio of large to small signal transconductance. It also gives the value of the amplitude of the oscillation. From this curve, we can see how the oscillation starts and how it stabilizes. At start up, the amplitude is very small, the gain is larger than required. The closed loop poles will be situated in the right half of the s-plane and we will have exponentially increasing oscillations. When the amplitude reaches the required one, equation (10) will be satisfied and this means that the poles will be on the $j\omega$ axis. Any further increase of the amplitude will decrease the gain and the poles will move to the left half of the s-plane and at that time the amplitude will decrease. So, we see that the amplitude will stabilize at the amplitude for which equation (10) is satisfied. An example of such oscillator is the Colpitts oscillator of lab #4.

Let us consider the following circuit: The transistors have $\beta = 100$.

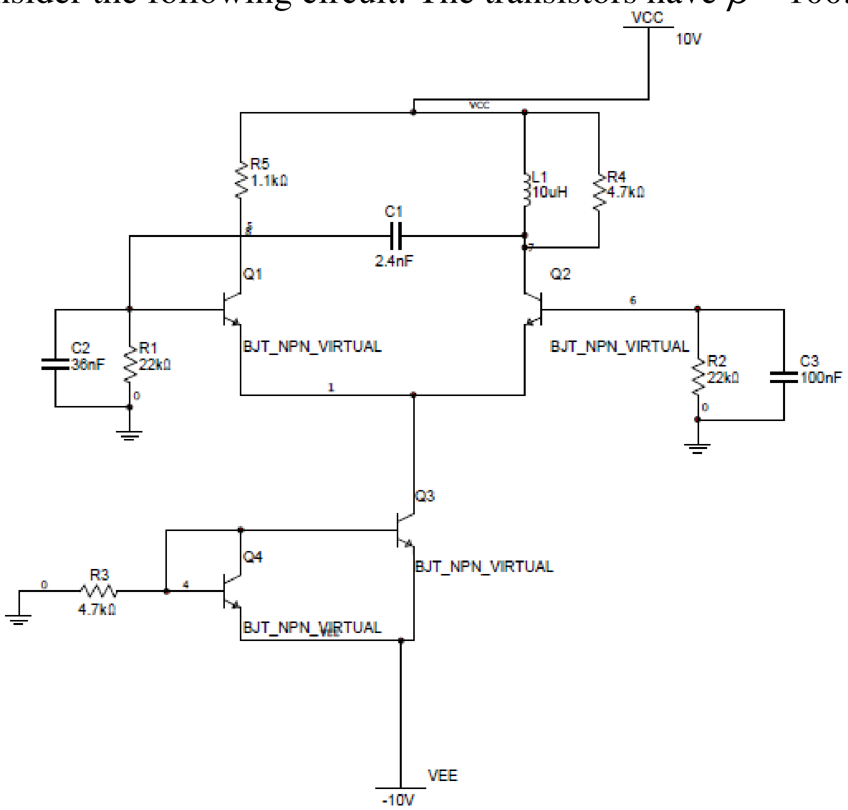


Fig. 4- 10 Differential amplifier Colpitts oscillator

It consists of an oscillator built around a differential amplifier. The coupling between the output (collector of transistor Q2) and the input is done using a split capacitor transformer-like network. As seen in chapter 2, if we assume that $nQ_{T'}Q_E > 100$ and $Q_E > 10$, we can transform the above circuit into the one shown below.

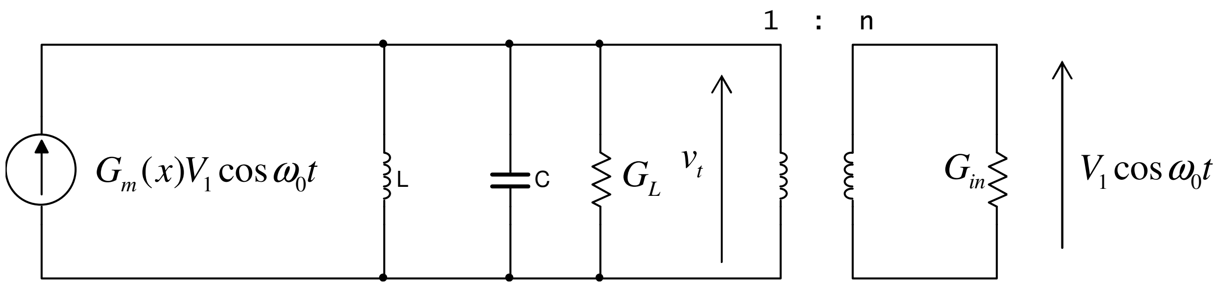


Fig. 4- 11 Equivalent circuit

The capacitance C is: $C = \frac{C1C2}{C1+C2} = 2.25\ \text{nF}$ so the resonant frequency is: $f_0 = \frac{1}{2\pi\sqrt{LC}} \approx 1\ \text{MHz}$. The input admittance of the transistor is: $G_{in} = \frac{G_m(x)}{\beta}$ as seen in chapter 3. We need the value of x in order to determine the value of the transconductance. However, we know that it is bounded by the value of the small signal one. In order to determine the small signal transconductance, we need to compute the biasing current.

$$I_k = \frac{V_{EE} - 0.75}{(2-\alpha)R3} \approx 2\ \text{mA}$$

The small signal transconductance is: $g_m = \alpha\frac{qI_k}{4kT} = \frac{2\ \text{mA}}{4\times 26\ \text{mV}} = 1.9\times10^{-2}\,\Omega^{-1}$ so the small signal input conductance is: $g_{in} = 1.9\times10^{-4}\ \Omega^{-1}$.

The turn ratio is: $n = \frac{C1}{C1+C2} = 0.0625$. We can now bound the value of $Q_{T'}$ and Q_E.

$Q_{T'} = \frac{\omega_0 C}{n^2 G}$. The conductance G is the sum of the input conductance and the conductance of the resistor R1.

$G \le \frac{1}{22\ \text{k}\Omega} + g_{in} = 2.3\times10^{-4}\,\Omega^{-1}$ giving $Q_{T'} \ge 16361$ and $Q_E \ge 1090$. The assumptions are largely verified. Furthermore, the very small value of G allow us to neglect the second term in equation (10). So, we can write:

$$\frac{G_m}{g_m} = \frac{G_L}{n g_m} = 0.17$$

So, using Fig.3-19, we obtain $x = 13.5$. With this value, the output fundamental current is given by $I_1 / I_k = 0.6366$ from Fig.3-18. So, the output voltage is:

$$v_t(t) = V_{CC} - R_L I_1 \cos\omega_0 t = 10\,V - (4.7\,k\Omega \times 0.6366 \times 2\,mA)\cos\omega_0 t = 10\,V - (5.98\,V)\cos\omega_0 t$$

The same analysis can be repeated for any LC oscillator built around a nonlinear controlled current source.

4.5 Stability factors

When we design and implement an oscillator, we must make sure that its parameters are not going to vary with different conditions such as aging, temperature, etc. The most important parameter that must be fixed is the frequency of oscillation. In general, frequency is allocated by government agencies to users and in order to avoid interferences, the frequency must not change with time and should not be affected by external conditions. We distinguish two different stability factors.

4.5.1 Direct stability factor

The direct stability factor indicates the sensitivity to a variation of the frequency setting elements of the circuit. We have seen that the frequency of many RC oscillators is inversely proportional to the product RC.

$$\omega_0 = \frac{k}{RC}$$

In this case, we can write:

$$\frac{\Delta\omega}{\omega_0} = -\left(\frac{\Delta R}{R} + \frac{\Delta C}{C}\right)$$

The above expression indicates that a relative variation of the frequency of the oscillator depends on the relative variation of the frequency setting elements. If these elements vary when external conditions such as temperature vary, we can appreciate the induced frequency variation and correct it. One commonly used technique is to select components with inverse temperature coefficients in order to have compensation. This technique is mostly used in LC oscillators. In this case, we have:

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ giving: } \frac{\Delta\omega}{\omega_0} = -\frac{1}{2}\left(\frac{\Delta L}{L} + \frac{\Delta C}{C}\right)$$

We can also put the oscillator inside a temperature stabilized oven in order to eliminate the temperature dependence of the circuit.

4.5.2 Indirect stability factor

When we design an oscillator, we assume that the elements are ideal. However, there are many parasitic elements that intervene in a circuit. The wires of the different components are inductive. There exists a distributed capacitance between the turns of an inductor and there exist many nonlinear memory elements that are present in active devices. We can model these elements as "spurious" poles and zeroes that are introduced in the open loop transfer function. In general, these spurious poles and zeroes have a very large frequency (imaginary part) and a very negative real part. We can express the real open loop transfer function as:

$$A_{Lreal}(s) = A_L(s)A_{Lspur}(s) \tag{11}$$

where $A_L(s)$ is the theoretical open loop transfer function, $A_{Lspur}(s)$ is the part of the transfer function due to the spurious poles and zeroes and $A_{Lreal}(s)$ is the actual open loop function. From the Barkhausen conditions, we know that the frequency of oscillation is given by the frequency for which the phase of the open loop transfer function is zero. From equation (11), we can write:

$$\phi_{real}(\omega) = \phi(\omega) + \phi_{spur}(\omega) \tag{12}$$

where $\phi(\omega) = \arg\left[A_L(s)\right]_{s=j\omega}$.

Consider the following phase responses:

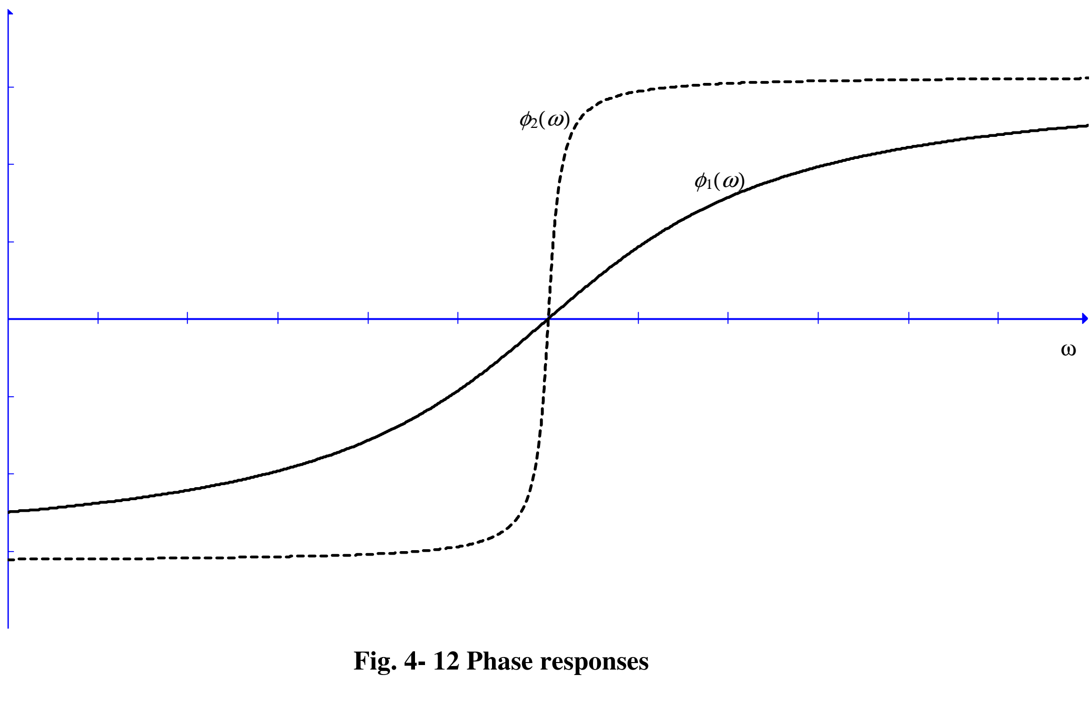


Fig. 4- 12 Phase responses

In Fig. 4- 12, we can observe two different theoretical phase responses that pass by zero at the same abscissa. According to equation(12), the actual phase response is going to shift up or down by a random small amount. It is evident that the intersection with the frequency axis is going to shift by a much smaller amount for the phase response $\phi_2(\omega)$ than for $\phi_1(\omega)$. So, the spurious poles and zeroes will have a much smaller effect on the frequency of oscillation for $\phi_2(\omega)$. From the figure, we can observe that the main difference between the two curves is their slope around the frequency ω_0. It is clear that the steeper the curve around ω_0, the less sensitive the circuit is to spurious poles and zeroes. So, we can define an "indirect frequency stability factor" from the phase response of the theoretical open loop gain. We define it as the ratio of the variation of phase over the relative frequency change around the frequency of oscillation.

$$S_F = \frac{\Delta\phi}{\left(\frac{\Delta\omega}{\omega_0}\right)} \tag{13}$$

From which we deduce:

$$\Delta\omega = \frac{\omega_0 \Delta\phi}{S_F}$$

The above relation indicates that the frequency shift is small if S_F is large. For example, a phase shift of 1° implies a frequency shift of 174.5 Hz around an oscillation frequency of 1 MHz if the value of the indirect stability factor has a value of 100.

In many cases, the variations are very small, so we can replace the variations by differentials, and we can write:

$$S_F = \omega_0 \left.\frac{d\phi}{d\omega}\right|_{\omega=\omega_0} \tag{14}$$

Equation (14) allows the computation of S_F from the expression of the open loop transfer function. For lumped circuits, the open loop transfer function is a ratio of real polynomials. This means that is has zeroes and poles that are either real or they occur as pairs of conjugate complex numbers. The open loop transfer function can be written as:

$$A_L(s) = A_0 \frac{(s-z_1)(s-z_2)\cdots(s-z_M)}{(s-p_1)(s-p_2)\cdots(s-p_N)}$$

For a real A_0, the phase of this transfer function is the sum of the phases due to the zeroes plus the sum of the phases due to the poles.

$$\phi(\omega) = \arg\left[A_L(j\omega)\right] = \sum_{k=1}^{M}\phi_{zk} + \sum_{i=1}^{N}\phi_{pi}$$

The phases introduced by the poles are negative because they occur at the denominator.

From equation(14), the stability factor can then be written as:

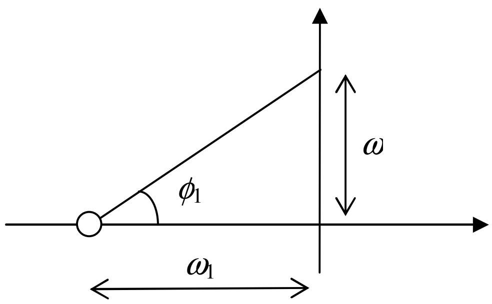
$$S_F = \sum_{k=1}^{M} S_{Fzk} + \sum_{k=1}^{N} S_{Fpk}$$

where $S_{Fzk} = \omega_0 \left.\frac{d\phi_{zk}}{d\omega}\right|_{\omega=\omega_0}$ and $S_{Fpk} = \omega_0 \left.\frac{d\phi_{pk}}{d\omega}\right|_{\omega=\omega_0}$.

We just have to know the contribution of a real zero (pole) and a pair of complex conjugate zeroes (poles) in order to be able to compute the stability factor for any transfer function. This computation is performed using a pole and zero plot.

Indirect stability factor corresponding to a real zero (pole):

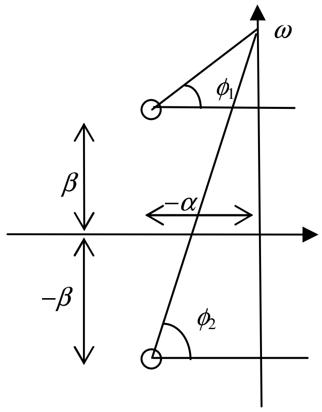
Consider a zero $z_1 = -\omega_1 \in \mathbb{R}$.



The angle ϕ_1 is given by: $\phi_1(\omega) = \tan^{-1}\left(\frac{\omega}{\omega_1}\right)$ and this gives a stability factor equal to:

$$S_{Fz} = \omega_0 \left.\frac{d\phi_1}{d\omega}\right|_{\omega=\omega_0} = \frac{\omega_0/\omega_1}{1 + \left(\omega_0/\omega_1\right)^2}$$

Let us now consider a pair of complex conjugate zeroes: $z_{1,2} = -\alpha \pm j\beta$. The corresponding plot is:



The total phase contribution corresponding to the pair of zeroes is given by:

$$\phi(\omega)=\phi_1+\phi_2=\tan^{-1}\frac{\omega-\beta}{\alpha}+\tan^{-1}\frac{\omega+\beta}{\alpha}$$

This gives an indirect stability factor of:

$$S_F=\omega_0\left.\frac{d\phi}{d\omega}\right|_{\omega=\omega_0}=\frac{2\alpha\omega_0\left(\omega_{res}^2+\omega_0^2\right)}{\left(\omega_{res}^2-\omega_0^2\right)+4\alpha^2\omega_0^2}\ \text{where}\ \omega_{res}^2=\alpha^2+\beta^2.$$

ω_{res} is the resonant frequency corresponding to the pair of complex zeroes. It is quite usual for an oscillator to operate at the resonant frequency. At that time, we have $\omega_0=\omega_{res}$ and the stability factor simplifies to:

$$S_F=\frac{\omega_{res}}{\alpha}=2Q_T$$

where Q_T is the "Que" of the pair of zeroes. If we consider poles instead, we have:

$$S_F=-2Q_T$$

Example: Consider the oscillator seen in section 4.2.

$A_L(s)=\dfrac{K}{(1+RCs)^3}$. It has 3 real poles at the position: $\dfrac{-1}{RC}$. So the indirect stability factor is: $S_F=\dfrac{-3\left(\omega_0/\omega_1\right)}{1+\left(\omega_0/\omega_1\right)^2}$ along with $\omega_0=\dfrac{\sqrt{3}}{RC}$ and $\omega_1=\dfrac{1}{RC}$. After replacement, we obtain: $S_F=\dfrac{3\sqrt{3}}{4}$. This is a quite small value. With a stability factor that small, it is very hard to adjust exactly the value of the frequency. Furthermore, any variation in the parasitic elements will have a large influence on the oscillation frequency.

The LC oscillators all have a transfer function having a zero at the origin and a pair of complex conjugate poles. This gives a stability factor equal to: $S_F = -2Q_T$. With typical capacitors and inductors, we can achieve maximum values of Q_T of 150. So, the largest value we can expect for the indirect stability factor is 300. This is much better than the RC oscillator seen before. We can improve the stability of the oscillator by having a very small direct stability factor. This can be achieved by selecting capacitors with a negative temperature coefficient equal in absolute value to the positive temperature coefficient of the inductor at the ambient temperature.

4.6 Crystal oscillators

In the previous section, we have seen that we can achieve indirect stability factors of about 300 with LC oscillators. This corresponds to a frequency shift of 58 Hz for a phase shift of 1° when the oscillation frequency is set to 1 MHz. However, this is the limit of what we can achieve with LC resonators. In order to achieve better results, we have to resort to mechanical resonators. A very high Q can be achieved with "crystal" resonators. It is possible to obtain Q's of about 10^4 at resonant frequencies around 1 MHz. This means that the indirect stability factor will be around 2×10^4. This corresponds to a frequency shift of 0.87 Hz for a phase shift of 1° (less than 1 Hz).So, if we put the oscillator circuit in a temperature stabilized oven, we can guarantee the frequency stability to the Hertz value when the oscillation is around 1 MHz. In fact, many test equipments have such oven stabilized references.

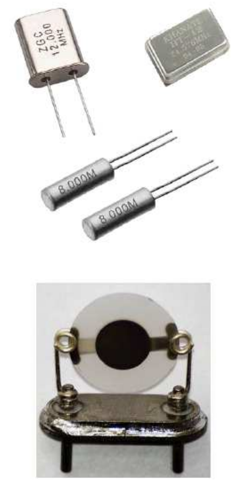


Fig. 4- 13 Crystal enclosures and mounting

Some materials such as quartz possess the piezoelectricity property. In other words, a quartz crystal submitted to a mechanical strain along a given axis will develop an electrical polarization along another axis. This process can be reversed. If we apply a voltage across this axis, the crystal will have a mechanical deformation along the first axis. This property is used to build transducers like the ones used in SONAR and in echographic imaging. However, in our case, we are interested in mechanical resonators. A crystal cut has some elasticity and a mass. So, it can act as a mechanical resonator. The piezoelectricity allows us to transfer this mechanical resonance to an electrical one.

We can show that a crystal has the following equivalent electrical circuit.

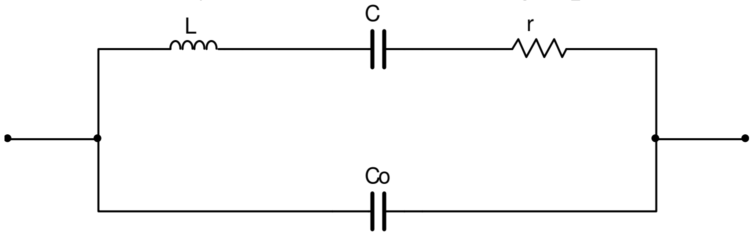


Fig. 4- 14 Crystal equivalent circuit

The inductance L is the electrical equivalent to the mechanical inertia. The capacitance C is the electrical equivalent to the elasticity and the resistance r is the electrical equivalent to the mechanical loss. These losses are greatly reduced by enclosing the piece of crystal inside a vacuum container. The capacitor C_0 on the other hand is due to the metallization on both faces of the crystal used to make the electrical connection. For example, typical values for an 8 MHz crystal are: $L = 14$ mH, $C = 27$ fF = 0.027 pF, $r = 8\ \Omega$ and $C_0 = 5.6$ pF. Except for C_0 and r, the other values are impossible to realize using discrete components.

The impedance of the circuit shown in Fig. 4- 13 is:

$$Z(s) = \frac{1}{C_0 s} \frac{s^2 + \frac{r}{L}s + \frac{1}{LC}}{s^2 + \frac{r}{L}s + \frac{C + C_0}{CC_0 L}} \qquad (15)$$

We introduce the following frequencies:

$\omega_0 = \frac{1}{\sqrt{LC}}$: It is the resonant frequencies of the series arm composed of L, C and r .We call it the resonant frequency.

$\omega_1 = \sqrt{\frac{C + C_0}{LCC_0}} = \omega_0 \sqrt{1 + \frac{C}{C_0}} \approx \omega_0 \left(1 + \frac{C}{2C_0}\right)$: It is the resonant frequency of a circuit composed of the parallel connection of L, C in series with C_0 and a transformed r. We call it the antiresonant frequency. From the above definitions of frequencies, we can also define the "pulling range" as $\Delta\omega = \omega_1 - \omega_0$. It is equal to:

$$\Delta\omega \approx \omega_0 \frac{C}{2C_0}$$

We can re-express equation (15) using the above notations:

$$Z(s) = \frac{1}{C_0 s} \frac{s^2 + 2\alpha s + \omega_0^2}{s^2 + 2\alpha s + \omega_1^2}$$

It has two finite zeroes at $z_{1,2} = -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2}$ and three poles at $p_0 = 0$ and at $p_{1,2} = -\alpha \pm \sqrt{\omega_1^2 - \alpha^2}$. The value of α is:

$$\alpha = \frac{r}{2L}$$

Using the fact that $\alpha = \frac{\omega_0}{2Q}$ for resonant circuits (see chapter 2), we obtain:

$$Q = \frac{L\omega_0}{r}$$

For the 8 MHz crystal described before, the different values are:

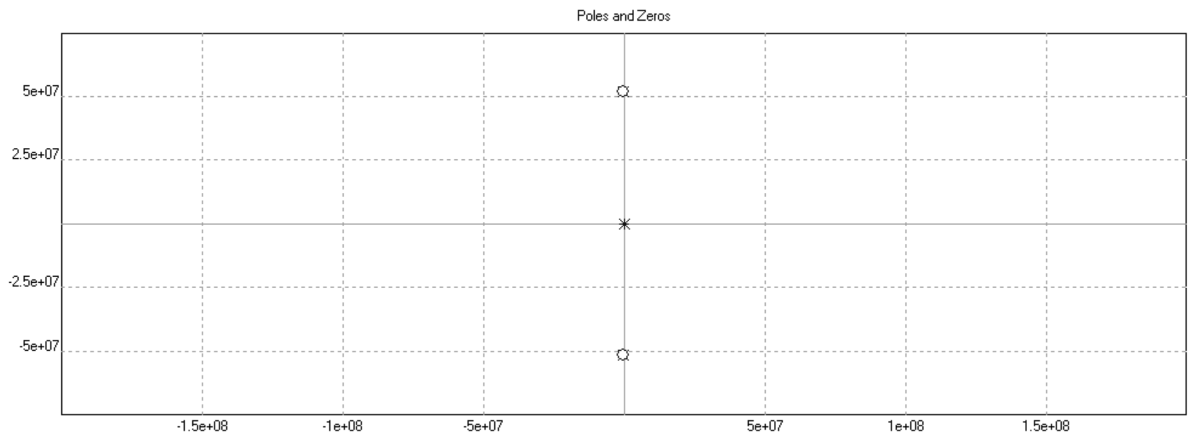
Resonant frequency: f_0 = 8.186046961 MHz

Antiresonant frequency: f_1 = 8.205757451 MHz

Giving a pulling range of Δf = 19.710490739 kHz

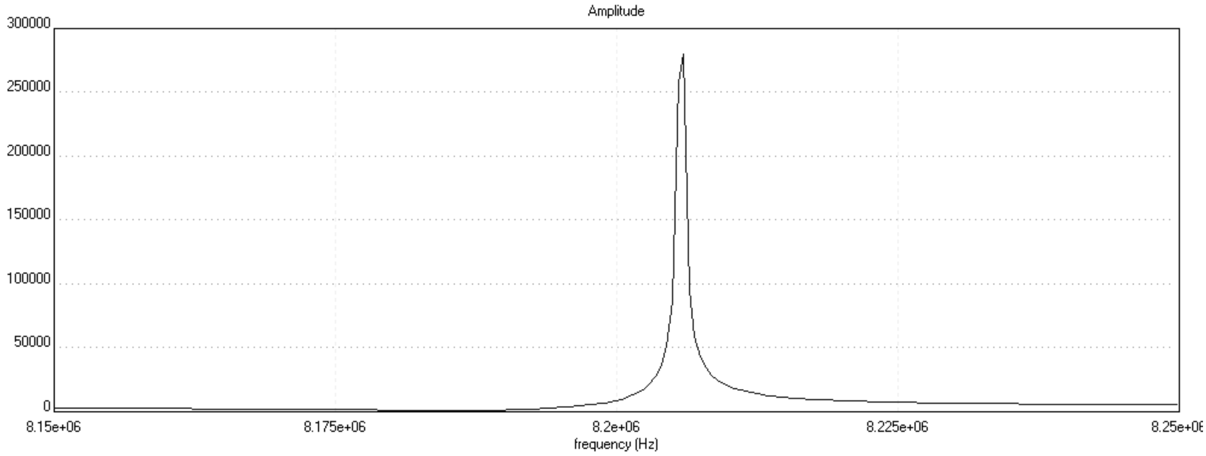
The Q corresponding to the complex zeroes or poles is: Q = 90010. With such high value of Q, the zeroes and the poles are practically:

$z_{1,2} = -\alpha \pm j\omega_0$ and $p_{1,2} = -\alpha \pm j\omega_1$. The corresponding pole and zero plot is:

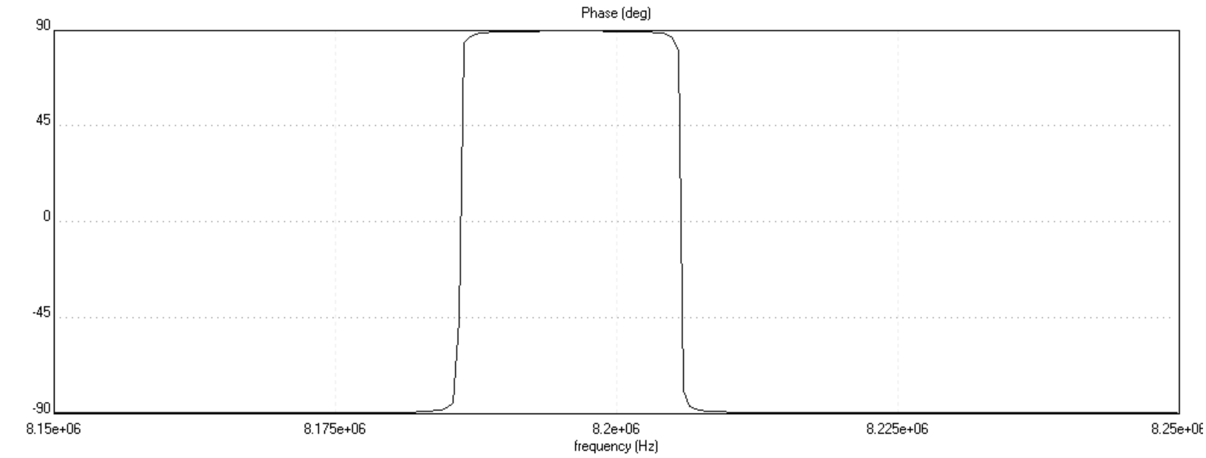


We remark that the complex poles and zeroes are practically superposed and they have a very small real part.

The magnitude of the impedance is shown below.



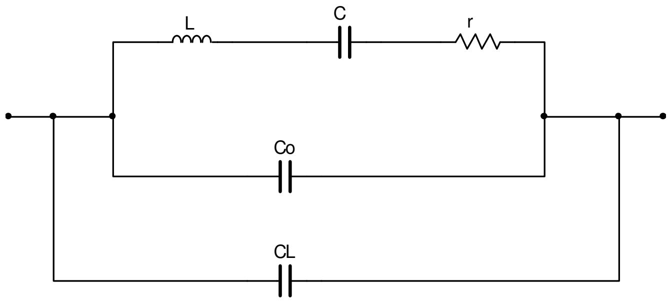
The phase of the circuit is:



The impedance is real at two frequencies. At the resonant frequency, the impedance is real and is minimum at a value of 8 Ω. At the antiresonant frequency, the impedance is also real and has a maximum value of 1.5 MΩ. At frequencies below the resonant frequency and above the antiresonant frequency, the crystal is capacitive. Between these two frequencies, the crystal is inductive. The value of the impedance at the antiresonant frequency can be computed easily from the pole and zero plot and is approximately given by:

$$R \approx \frac{1}{C_0 \omega_1} \sqrt{1 + Q^2 \frac{C^2}{C_0^2}}$$

When we want to use a crystal in an oscillator, we can operate it at the resonant frequency ω_0. In this case, we say that we have a series mode oscillator. The impedance of the crystal at this frequency is resistive and is minimal. In some other cases, we can operate the oscillator at a frequency between ω_0 and ω_1. This type of oscillator is called a parallel mode oscillator. In this case, the crystal is inductive and we must use a capacitance C_L in parallel with the crystal. In general, crystals are designed to be used at given particular frequency. In this case, the capacitance is specified. The frequency of oscillation is the parallel resonant frequency given by the following circuit:



We see that C_0 is replaced by C_0 in parallel with C_L. This gives an oscillation frequency of:

$$\omega_L \approx \omega_0 \left(1 + \frac{C}{C_0 + C_L}\right)$$

A typical parallel mode oscillator is the Pierce oscillator. Its circuit is shown below.

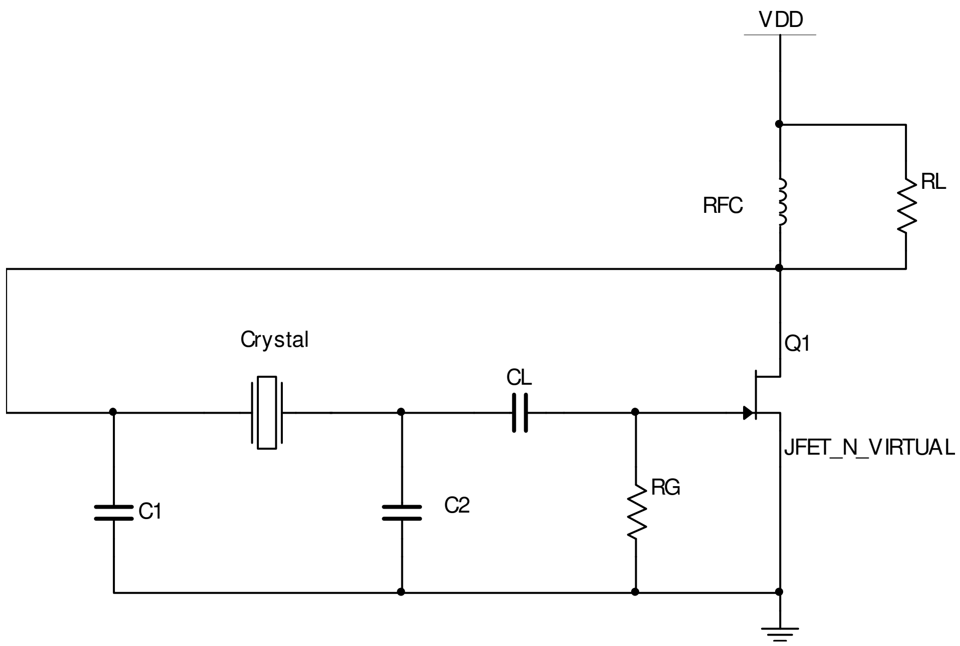
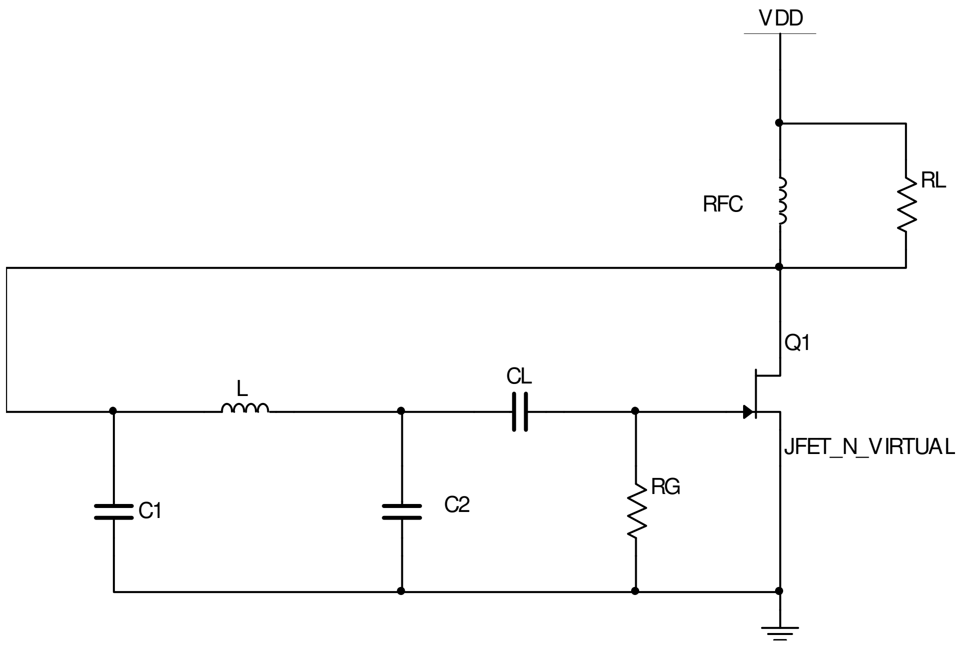


Fig. 4- 15 Pierce Oscillator

Before starting the analysis of the circuit, we must remark that the FET is clamp biased by the pair C_L and R_G. The RFC is an inductor that has a very high value. It is used to bypass the dc current and acts as a very large impedance for the ac current in the drain circuit.

Another point worth noting is that the circuit composed of the crystal and the two capacitors C_1 and C_2 must produce a phase shift of 180° at the frequency of oscillation to satisfy the phase requirement of the Barkhausen conditions. In order to produce such phase shift, the crystal must be inductive. This means that the frequency of oscillation is going to be inside the pulling range of the crystal, i.e. between the resonant and the antiresonant frequencies. So, we can replace the crystal by an inductance in the circuit of Fig. 4- 15.



In this part, we are going to perform a small signal analysis of the above circuit. The small signal equivalent circuit is shown below (Fig. 4- 16).

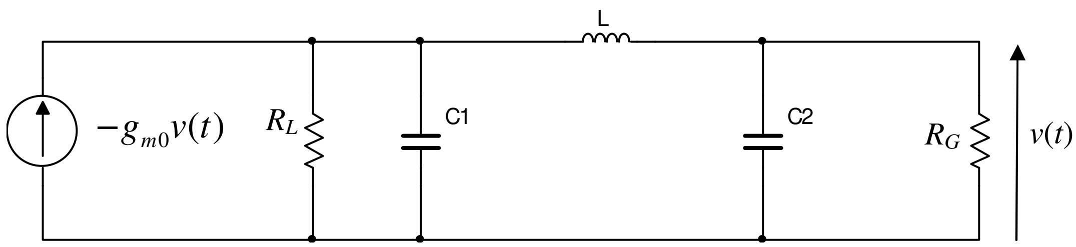
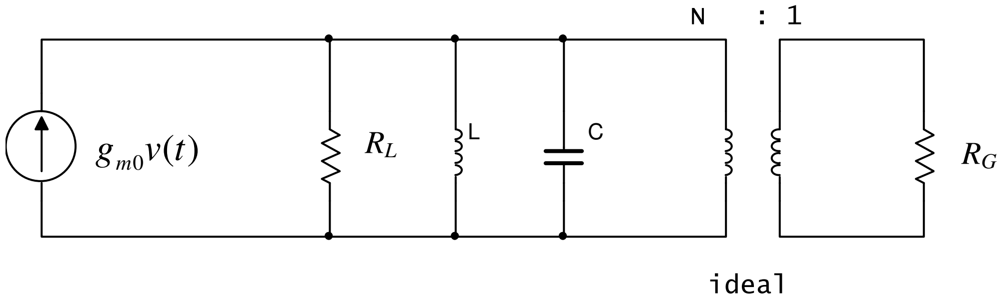


Fig. 4- 16 Small signal equivalent circuit

We have already studied the coupling circuit formed by the two capacitances C_1 and C_2 and the inductance L at the end of chapter 2. It is the "pi" circuit. Using the equivalence derived in chapter 2, we can replace the above circuit by:



where $C = \dfrac{C_1 C_2}{C_1 + C_2}$ and $N = \dfrac{C_2}{C_1}$

The voltage at the primary is: $\dfrac{g_{m0}v(t)}{\dfrac{1}{R_L} + \dfrac{1}{N^2 R_G}}$ and at the secondary, we must have this voltage divided by *N*. So, the voltage at the secondary must be: $\dfrac{g_{m0}v(t)}{\dfrac{1}{R_L} + \dfrac{1}{N^2 R_G}} \times \dfrac{1}{N}$. This voltage must be larger than $v(t)$ at the frequency of oscillation in order to have closed loop poles on the right half of the s-plane. This implies that:

$$g_{m0} \geq \frac{N}{R_L} + \frac{1}{NR_G} = \frac{C_2}{C_1}\frac{1}{R_L} + \frac{C_1}{C_2}\frac{1}{R_G}$$