Some exercises.

Pb.1

Consider the signal $s(t) = \text{sinc}^2(t)$ and the bandpass signal $x(t) = s(t)\cos 2\pi f_0 t - \hat{s}(t)\sin 2\pi f_0 t$ with $f_0 > 1$.

a) Derive the complex envelop of $x(t)$ in both the time and frequency domain.

b) Sketch the spectrum of $x(t)$.

c) Compute the energy of $x(t)$.

Pb.2

Consider the signal: $x(t) = \cos 2\pi 100t$. This signal is sampled with a sampling frequency $f_s = 120\text{ Hz}$. The output of the sampler passes through an ideal low pass filter with a cut off frequency $f_c = 60\text{ Hz}$.

Compute the output of the low pass filter.

Pb.3

The signal $s(t) = 3K\left(\cos 8\pi t + 2\cos 20\pi t\right)$ is input to an AM transmitter with a modulation index $m = 1$ and a carrier frequency $f_0 = 1000$.

a) Find K so that $s(t)$ is properly normalized.

b) Draw the positive frequency line spectrum of the modulated wave.

c) Compute the efficiency coefficient.

Pb.4

Consider the following signal:

$$\tilde{s}(t) = A\cos\omega_1 t + B\cos\omega_2 t$$

It is used to VSB modulate a carrier at the frequency $f_0 >> f_1$ and $f_0 >> f_2$.

The VSB filter has the magnitude response plotted in the text-book at page 104, fig.3.9. It is characterized by:

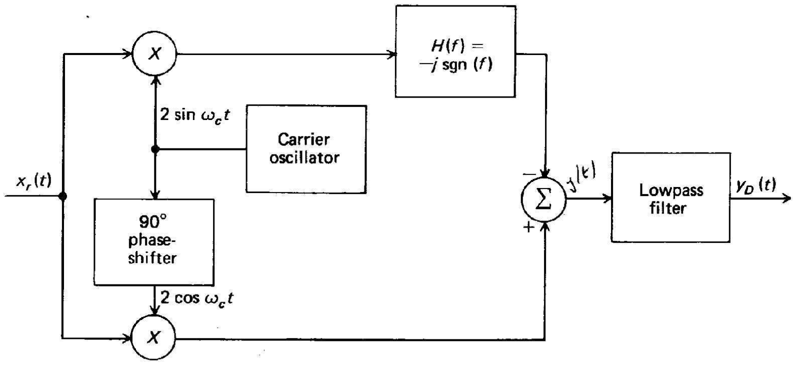
$$H(f_0 - f_1) = \varepsilon e^{j\phi}$$
$$H(f_0 + f_1) = (1-\varepsilon)e^{j\theta_1}$$
$$H(f_0 + f_2) = e^{j\theta_2}$$

The signal is to be coherently demodulated.

1. Compute the complex envelop of the VSB signal.
2. Using the results in your course notes, derive the expressions for θ_1 and θ_2 as a function of ϕ in order to have a correct output from the demodulator.

Pb.5

a) Show that the following system can be used to demodulate a lower sideband SSB signal.



The above system is known as a *phase shift demodulator*.

b) Show that the above system can be modified to yield an upper sideband SSB phase shift demodulator.

c) Describe the output of the lower sideband phase shift demodulator if we apply at the input an upper sideband SSB signal at the same carrier frequency.

Pb.6

a) Consider the signal $s(t) = \cos\omega_m t$ and let $\omega_0 >> \omega_m$ be a carrier.

Show that the AM signal modulated by $s(t)$ consists of the sum of three sine waves, the DSB-SC signal is the sum of two sine waves and the USB-SSB signal is a single sine wave.

b) Consider now the following signal:

$$\tilde{s}(t) = 7\cos\omega_m t + 3\cos 3\omega_m t + \cos 5\omega_m t$$

We use $\tilde{s}(t)$ to produce a 30% modulated AM waveform with a carrier $\omega_0 \gg \omega_m$. Compute the efficiency of the AM signal.

Pb.7

Consider the following signal:

$$x(t) = 2(1 + 0.6\cos\omega_m t)\cos\omega_0 t$$

where $\omega_m \ll \omega_0$. It is applied to a band pass filter with transfer function:

$$H(\omega) = \frac{1}{1 + j\dfrac{\omega - \omega_0}{\alpha}} \qquad \omega > 0$$

Compute the output of the band pass filter if $\omega_m = \alpha$.

(Remember that the envelop of the signal contains a dc component).

Pb.8

Consider the band pass signal:

$$x(t) = s_1(t)\cos(\omega_0 - \omega_m)t + s_2(t)\cos(\omega_0 + \omega_m)t$$

where $\omega_m \ll \omega_0$ and $s_1(t)$ and $s_2(t)$ are low pass signal band limited to $W \ll f_0$.

1) Express the complex envelop of the signal in both quadrature form (real and imaginary) and in polar form (modulus and phase).

2) Simplify $R(t)$ and $\varphi(t)$ assuming $|s_1(t)| \gg |s_2(t)|$.

Pb.9

The signal $s(t) = \mathrm{sinc}\,40t$ is to be transmitted in AM with a modulation index $m < 1$.

1) What is the minimum value of the carrier frequency?

2) Sketch the spectrum of the modulated signal.

3) Repeat the same problem for USB-SSB.

Pb.10

1) Compute the power and the RMS value of a square wave that has a peak to peak value of 2 V and a DC value of 1V.

2) Repeat the question for a sine wave that has the same DC and peak to peak values.

Pb.11

Consider a real bandpass filter with the transfer function $H_{bp}(f) = |H(f)|\exp[j\varphi(f)|$ such that:

$$|H(f)| = K_0 + \frac{K_1}{f_c}(f - f_c) \quad ; \quad \varphi(f) = 0 \;;\; 0 < f_l < f < f_u$$

where $K_0 > \left(K_1 / f_c\right)(f_u - f_l)$.

a) Sketch $H_{bp}(f)$ taking $f_l < f_c < f_u$.

b) If the input of the signal is $x(t) = A(t)\cos 2\pi f_c t$, compute the quadrature components of the output signal $y(t)$.

Pb.12

The signal $\tilde{s}(t) = \frac{1}{2}\cos 2\pi 70t + \frac{1}{3}\cos 2\pi 120t$ is added to a carrier signal $c(t) = \cos 2\pi f_0 t$, $f_0 = 10\,kHz$ and presented at the input of a square law device with a transfer function $y = a_1 x + a_2 x^2$.

a) Give the center frequency and bandwidth of the filter such that this system will produce a standard AM signal.

b) Determine values of a_1 and a_2 such that $A = 10$ and $m = 0.5$.

Pb.13

Let $x(t)$ be an arbitrary function, and let $x_c(t)$ be its “causal” part; that is:

$$x_c(t) = \begin{cases} x(t) & t \ge 0 \\ 0 & t < 0 \end{cases}$$

a) Show that $x_c(t)$ can be written as:

$$x_c(t) = \frac{1}{2} x(t)\left[1 + \operatorname{sgn} t\right]$$

b) Compute $X_c(f) = \mathcal{F}\left[x_c(t)\right]$ in terms of the Fourier transform of $x(t)$.

c) Let $x_c(t) = x(t)$; that is, $x(t) = 0$ for $t < 0$. Derive a relation between the real part and the imaginary part of $X(f)$.