

# Distance Protective System Performance Enhancement Using Optimized Digital Filter

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*Abstract:* - In conventional transmission line protection, a three – zone stepped directional distance system is used to provide the primary as well as remote backup protection. The voltage and current phasors needed by the distance relay for determining the impedance may be measured with integrated Phasor Measurement Unit. However, accuracy of this measurement may be affected by the power disturbances such as power swings and switching actions resulting in fast and slow DC offsets decaying and harmonics, etc. The quality of this measurement may cause mal-operation of distance relays which in turn may affect the reliability of the whole protective scheme. To mitigate these effects for improving the quality of measurements and hence the distance relay performance, this work proposes a new real-time digital filtering method for removing the unwanted DC offset and harmonics and hence improving SDFT algorithm. To validate the present method, the performance of developed distance relay is tested using signal generated by Simulink/MATLAB simulator under different conditions. The obtained simulation results are satisfactory.

*Key-Words:* - Digital distance relay, PMU, power grid, DC offset, digital filter

## 1 Introduction

Power system protection is the process of making the production, transmission, and distribution of electrical energy as safe as possible from the effects of failures and events that place the power system at risk. When the faults occur in such power system, protection systems are required to isolate faulted part of the power system, and leave the healthy parts of the system connected in order to insure the continuity of the power supply. The operational security of the power system depends upon the successful performance of the thousands of relays that protect equipments and hence protect the whole system from cascading failures. Thus, the failure of a relay to operate as intended may jeopardize the stability of the entire system and equipment in it. The mal-operation of this relay is generally due to unnecessary tripping that reduces the security of such system and hence its reliability. In order to avoid the unnecessary tripping, many techniques have been developed such as digital filter or blinder.

Accurate and fast measurement of the voltage and current phasors of the fundamental components is very important in three-phase distance relay that may be investigated by an integrated Phasor measurement unit (PMU). In digital relaying system, discrete Fourier transform (DFT) is the most widely used filtering algorithm [1-3] for computing the fundamental phasors and their symmetrical components. However, some

transient disturbance currents of a transmission line may contain a DC offset which decays exponentially with time (time constant of the line inductance to resistance ratio  $L/R$ ), or a large number of unwanted harmonics or decaying DC components due to the thyristor-controlled switched capacitor (TCSC) compensated lines [4]. This latter always needs few cycles for decaying DC component or 10– 20 cycles for harmonics component to obtain the accurate fundamental phasors by discrete Fourier transform DFT algorithm. From the evaluation performed using the ideal network [5], the DC offset may have an effective impact on the Fourier algorithm and if no correction is applied, the relative error of the real amplitude from the Fourier algorithm may reach 20%, which purely caused by this decaying offset. For a high performance control and protection applications such a large relative error is not allowed. The performance of the techniques employed directly determine the functions of this equipment and affect their behaviors under various service conditions. Hence, the real-time accurate phasor measurement of the fundamental component and/or symmetrical components is essential and crucial to the safe and economic running of modern electric power systems [6].

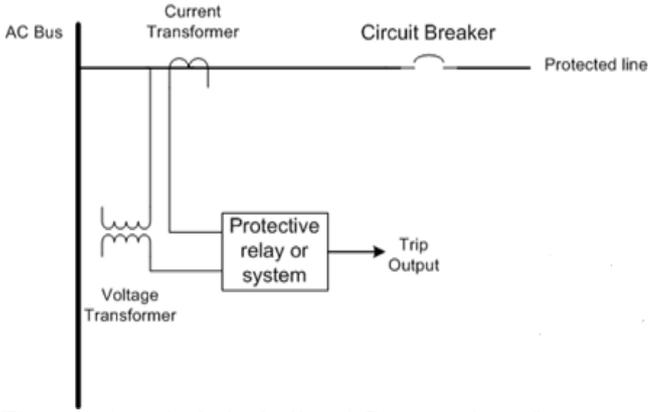


Figure 1. A typical single line AC connection of a protective distance relay.

## 2 Distance Relay

Protections based on distance relaying have been generally used in the power grid and especially in transmission lines in order to detect the fault rapidly and disconnect the faulted part only. This maintains a reliable operation of the power system and to ensure continuity of power transmission.

The basic principle governing the operation of a distance relay is the ratio between the voltage  $V$  and the current  $I$  at the relaying point as shown in Fig.1. The ratio  $V/I$  represent the measured impedance  $Z_f$  of the faulty line between the relay location and the point of fault occurrence. Then the measured impedance is compared to the set impedance, and if this  $Z_f$  is within the reach of the relay then the fault will be cleared. However, these two dynamics (current and voltage) may be affected by the DC offset and harmonics. In some numerical relays, DFT algorithm may be used to filter these unwanted distortions that may appear in the measured data. In the others, filters have introduced before the impedance determination takes place. However, traditional relays and previous numerical relays will wait few cycles for taking decision until all unwanted distortions disappear. In our approach, an optimized digital filter has been used which leads the relay to trip rapidly with good accuracy as shown in Fig.2.

Digital distance relay has been implemented using phase comparators of two input quantities for obtaining the operating characteristics which may be circles when plotted on an R-X diagram. For the impedance elements of a distance relay, the two quantities being compared are the voltage and current measured by the relay.

The signals  $S_1$  and  $S_2$  are given to the phase comparator in the following form [7]:

$$S_1 = -KI + K_2|V|\angle(\theta - \varphi) \quad (1.a)$$

$$S_2 = KI + K_4|V|\angle(\theta - \varphi) \quad (1.b)$$

Substituting in the general equation of phase comparator at threshold we obtain:

$$-K^2I^2 + [-KK_4 \cos(\theta - \varphi) + K_2K \cos(\theta - \varphi)]VI + K_2K_4V^2 = 0 \quad (2)$$

Letting  $\frac{V}{I} = Z$ , we get:

$$-K^2 + K_2K_4Z^2 + KZ(K_2 - K_4) \cos(\theta - \varphi) = 0 \quad (3)$$

Replacing  $Z^2$  by  $R^2 + X^2$  and after some algebraic manipulations, we finally obtain:

$$\left(R - \frac{K'(K_2 - K_4)}{2K_2K_4} \cos\theta\right)^2 + \left(X - \frac{K(K_2 - K_4)}{2K_2K_4} \sin\theta\right)^2 = \left(\frac{K(K + K_4)}{2K_2K_4}\right)^2 \quad (4)$$

This equation represents a circle with a radius of  $\left(\frac{K}{2K_4} + \frac{K}{2K_2}\right)$  and centered at  $\frac{K(K_2 - K_4)}{2K_2K_4} \angle\theta$  on the R-X plan.

Figure 2 shows the characteristic of the offset mho relay.

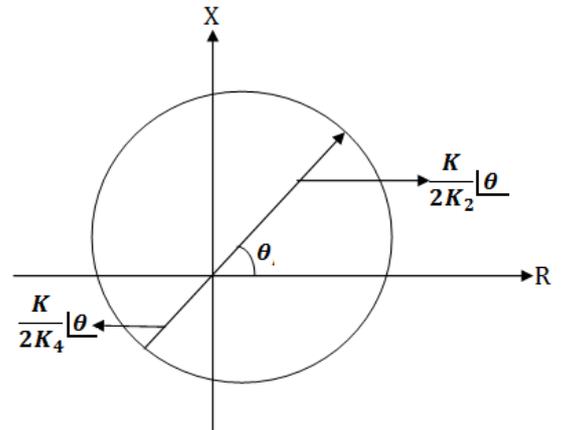


Figure 2. The characteristic of the offset mho relay.

## 3 DC Offset

For an ac input signal that is associated with a DC offset component, a constant DC and exponentially decaying signal, Gu and Yu [8] propose a modified Fourier filter algorithm using a data window of one cycle plus two samples to compute and perform compensation to remove the unwanted DC offset. The idea behind this algorithm is that the decaying component can be completely removed from the original signal once its parameters are determined. The weakness of the proposed algorithm is that more calculation is needed for eliminating the DC offset. The data window is relevant when implementing this algorithm for the real-time application. A digital mimic filter has been proposed [9], to suppress the effect of an exponentially decaying component over a wide range of time constant (0.5 to 5 cycles and larger) and then apply the DFT algorithm to compute the phasors. A good performance has been obtained with this mimic filter when its time constant is

identical to the time constant of the exponentially decaying DC component. Another way where the Taylor series expansion is used to approximate the decaying direct component, then the fundamental phasors are estimated by means of curve fitting technique, using least error squares [10]. To enhance the computation speed, the recursive least squares computation curve fitting algorithm is introduced [11]. In addition, a DFT algorithm and least error square technique are combined to estimate the phasor without DC offset signal. Another method was proposed to identify the magnitude and the time constant of the decaying DC offset component [12]. In this method, the residual terms caused by some harmonics are ignored in the estimation procedure. The assumption that these residuals are negligible should not be taken for granted, and needs to be investigated further [5]. The performance of Kalman filters is evaluated in [9]. It was concluded that the third-order Kalman filter is sensitive to variations of the DC offset time constant. A kalman filter should only be superior in removing the DC-offset if its time constant is the same as one modeled in the state transition matrix [9].

This work proposes a method that can correctly extract the phasors of the fundamental components as well as symmetrical components from voltage or current waveforms and then estimate their instantaneous amplitude, phase angle, and frequency with good accuracy and in real-time, even when disturbances occur in large scale and complex power systems. The proposed algorithm is a real-time processing system since a sample by sample basis instead a frame or cycle basis (data window) to obtain the accurate fundamental phasors. This is to fulfill the high speed measurement and detection feature required by the PMU and protective system [13,14]. The approach consists first of removing unwanted dc components of the input measured signal using a fast digital filter algorithm, which is suitable for such a real-time application, and then provide the filtered signal to the Smart DFT[15] algorithm to accurately generate the filtered phasor measurement components.

The ideal network shown in Fig.3 may be used to reveal how the source of the decaying DC offset that is induced in the current. In the network, the  $L/R$  is variable according to the power system operation conditions and the fault is variable as well. Assume the switch  $k$  is closed at  $t=0$ , and applying the Kirchoff voltage law leads to the following differential equation:

$$V_m \sin(\omega t + \varphi) = Ri + L \frac{di}{dt} \quad (5)$$

By solving Eq.(5), the current  $i(t)$  will be as follows:

$$i(t) = -\frac{V_m e^{-\frac{R}{L}t}}{\sqrt{R^2 + (L\omega)^2}} \sin\left[\varphi - \tan^{-1}\left(\frac{L\omega}{R}\right)\right] + \frac{V_m}{\sqrt{R^2 + (L\omega)^2}} \sin\left[\omega t + \varphi - \tan^{-1}\left(\frac{L\omega}{R}\right)\right] \quad (6)$$

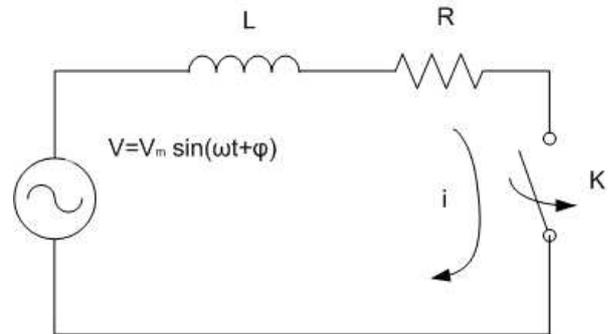


Figure 3. Ideal network

As seen in Eq.(6), the first term is a transient decaying offset component that is function of the parameters  $R$ ,  $L$ ,  $\omega$ ,  $\varphi$  and  $V_m$ . The effect of this term has been evaluated in reference [16] and concluded that the DC offset may have a severe effect on the Fourier algorithm. The relative error of the amplitude from the Fourier algorithm may reach 20% due to this decaying DC offset.

## 4 Optimized Digital Filter

Many filters have been proposed to eliminate the DC offset harmonics from the waveforms. A high pass filter with small cut-on frequency as compared to the Nyquist frequency is a required specification for the optimum design of the digital filter. As the sampling rate is high as compared to a required cut-on frequency, finite impulse response (FIR) standard design leads to very long filter length and computations [17, 18]. In order to reduce the memory length and computation effort, to suit the high speed real-time system, an infinite impulse response (IIR) recursive filter that achieve extremely low cut-on frequencies is used.

A digital filter based solution is proposed to remove the unwanted disturbances using digital filter design techniques. In addition, the required property of a measuring algorithm in protective relaying is to trace a given feature of signal (such as magnitude and phase) that may serve as a initial quantity for certain operating principles, which changes abruptly due to faults in a power system.

The present filtering application imposes different kind of specifications. In one hand, the time domain requirement were both a high speed and accurate system response are needed. In the other hand, the frequency domain requirements (DC, sub-synchronous and harmonic components) which are the magnitude response within small bandwidth and sharp frequency

edges as well as an approximately constant group delay in this band are required too.

If unusual amplitude or phase responses or delay characteristics are required, then the only available approach for the design of recursive filters is through the use optimization methods. In these methods, a discrete-time transfer function is formulated on the basis of some desired amplitude or phase response or some specified group-delay characteristic. A norm of the error function is then minimized with respect to the transfer-function coefficients. Then the design of this filter needs a simultaneous multi-objectives optimization. Usually the best optimum value of all the objective functions of this filter design can be obtained for some values of design variables. A compromise or a trade-off between the objective functions must be used to achieve a satisfied filter design.

Multi-objective optimization can be represented mathematically by minimizing a vector of objective functions that is subject to a set of constraints. This can be generally stated as:

$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \psi(x) = [\varphi_1(x), \varphi_2(x), \dots, \varphi_n(x)] \quad (6)$$

Subject to

$$\begin{aligned} \lambda_i(x) &= 0, \quad i = 1, 2, \dots, m \\ \beta_k(x) &\leq 0 \quad k = 1, 2, \dots, r \\ x^l &< x < x^u \end{aligned}$$

where  $x$  is the vector design parameters,  $\psi(x)$  is the vector of objective functions of length  $n$ ,  $\lambda_i(x)$  and  $\beta_k(x)$  are vector of length  $m$  containing the values of the equality and inequality constraints evaluated at  $x$ , and the solutions  $x$  is bounded by the lower ( $x^l$ ) and the upper ( $x^u$ ) limiting variables.

The considered recursive digital filter requirements can be designed using an optimizing approach that satisfies or minimizes three multi-objective functions. These functions are based on

- 1) specified or a desired magnitude response specification,
- 2) an approximately constant group delay with a minimum phase shift, and
- 3) a minimum time response or settling time.

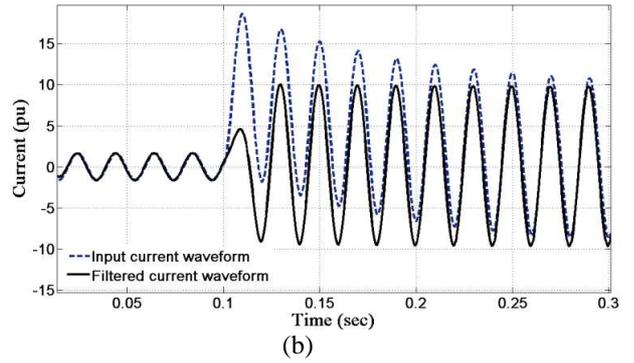
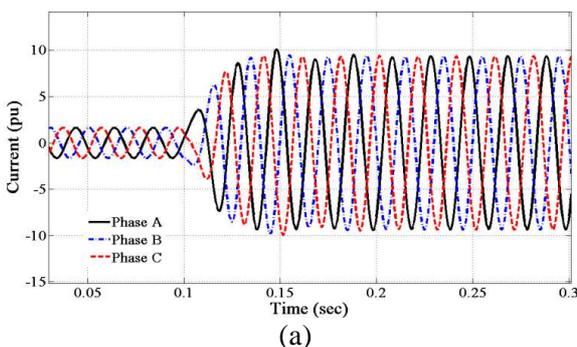


Fig.4 Current waveforms for a fault:  
(a) three phase current output of the ODF,  
(b) ODF input/output of a-phase current waveform

The optimization approach considers the discrete-time transfer function which is formulated on the basis of some desired amplitude response and a stability margin  $\delta$  parameter. A norm of the error function is then minimized with respect to the transfer-function coefficients with a prescribed  $\delta$ . The stability margin parameter  $\delta$  is varied to optimize the filter coefficients which minimizes mainly the magnitude response, satisfies the best approximately constant group delay and also minimizes the mean square error of the filter time response (minimum settling time). The Frequency domain objective function that is the magnitude frequency response based objective function which is first formulated by considering the stability parameter, and then the phase response or the group delay objective function is derived as well. The time domain objective function is synthesized by aggregating many time domain system performance indicators into a normalized mean square error obtained from the desired or an idealized step response and the optimized filter step response. Figure 4 shows the optimized digital filter (ODF) input/output current waveforms for a fault.

## 5 Simulation

### 5.1 Circuit Description

The experimental setup for testing distance relay using Simulink model is illustrated in Fig.5. After implementing this system using Simulink, we have run the simulator for 1 sec and the fault appears at 0.01 sec at different distances from the relay location.

### 5.2 Simulation Results and Discussion

The above system was simulated in MATLAB using the SimPower system toolbox of SIMULINK to test the mho distance relay performance. The mho distance relay characteristics and the fault impedance as well as the output trip signal of the relay are displayed as shown in Fig 6 and Fig.7 respectively.

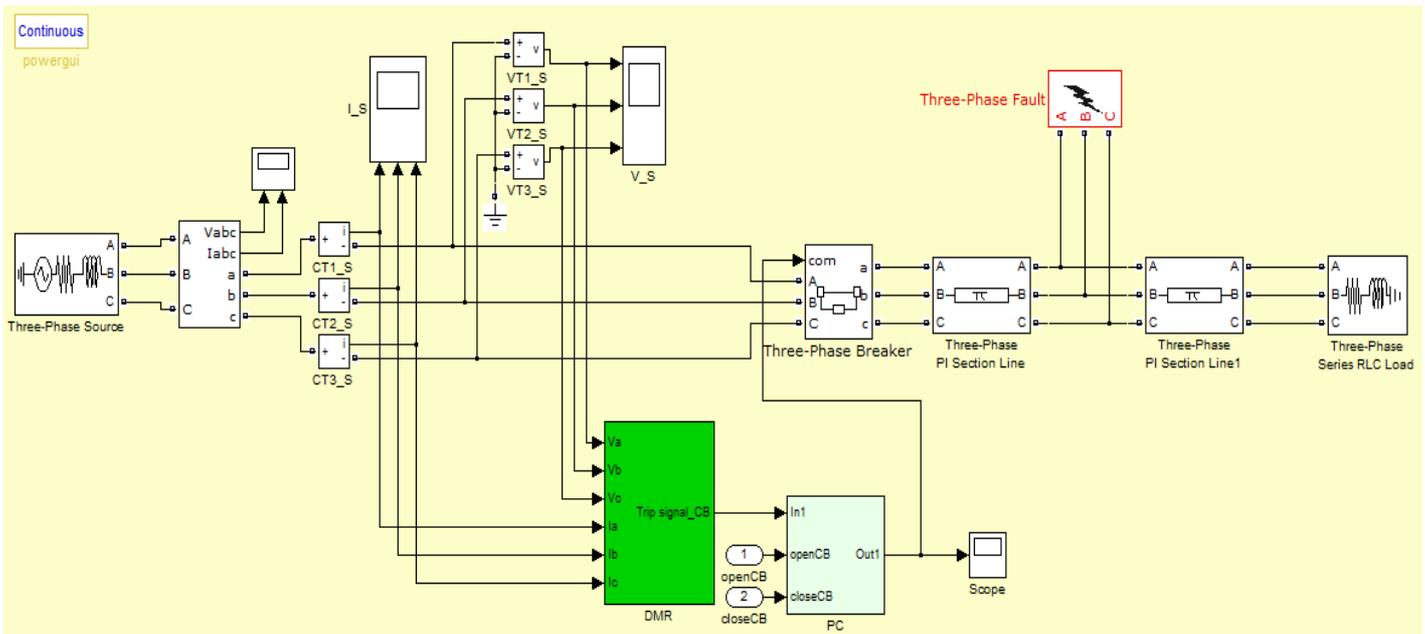


Fig.5 Simulink model of experimental set up for the relay tester

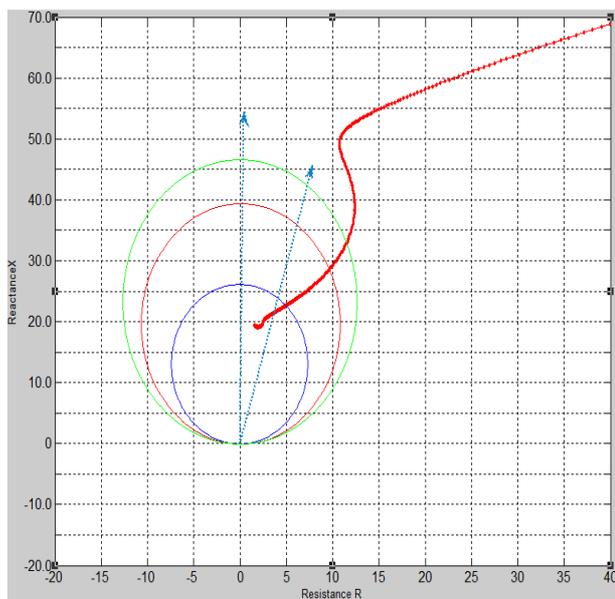


Figure 6. The mho distance relay characteristic and the fault impedance.

## 6 Conclusion

The results of the simulation and testing of the developed distance mho relay model for a simple transmission line model built in Simulink. Figure 6 represents phase A-to-ground fault (a-g) applied at different distances and this result will be the same for the other cases where the fault is applied to phase-b or phase-c (b-g or c-g). The obtained simulation results show that the used method is capable of completely eliminating the dc offset and hence greatly improving the reliability of the full-cycle DFT algorithm. Moreover, the performance of the proposed algorithm has been tested under transient and dynamic power system conditions, which are important for the protective relaying applications. It can be noticed that the digital filter that is appropriate to digital relay tests results are satisfactory.

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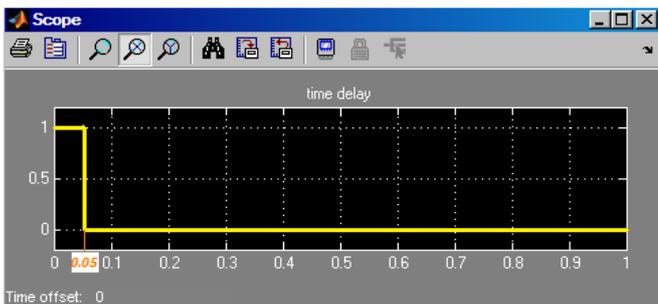


Figure 7. The output trip signal of the distance relay.

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