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Meteorological satellite images modeling using autoregressive process

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Abstract
The objective of this work is a modeling of the satellite images precipitations (or rainfalls) that are useful for hydrologic applications and an investigation on a possible different description of precipitations over the sea and on the continent are attempted in the satellite images concerning the Mediterranean region. The model used in this paper is the autoregressive process AR (p), where p is its order. This model makes possible the description of a series of 2208 satellite images, recorded during a period of 3 months via the Meteosat channel IR03.9.
To conduct a detailed study of the phenomenon, a scanning area of 20×50 pixels in the two areas (sea and continent) is adapted to examine all precipitation areas on the images, using a window (5X5 pixels) and a pixel-by-pixel processes.
The order of the AR model found in this paper, for the Mediterranean region, is “1”, which is invariant in time and space.

Keywords: *Images, Modeling, precipitations, rainfalls, Meteosat, Aautoregressive, Satellite.*

1. Introduction
Over the past decade, great attention has been paid to the parameterization of the rainfall fields. In general, the most successful hydrological models are those that combine spatial and temporal variations of precipitations.[6]
Analyze a chronological series consists in finding an adequate mathematical model of the series evolution mechanism. The obtained model is used to fulfill the objectives such as prediction and control.
For a series which collect the satellite rainfall estimation, the methods published in literature are mainly based on the determination of precipitation rates associated with different types of clouds, or the characterization of atmospheric convection, or tracking the evolution of cloud [3] or evaluating the frequency of occurrence of cold cloud [5]. However, satellite images do not ensure accurate identification of the cloud masses [1].
In our case, we used the evaluation of precipitation in the satellite images.

The remainder of the paper is organized as follows: Section (2) focuses on the mathematical formulation of an autoregressive process, sections (3) and (4) deal respectively with the database used in this work and the data processing. We illustrate and validate different results in sections (5) and (6). Finally, and in section (7), we give our conclusion.

2. Mathematical formulation

2.1. Autoregressive process of order p
Autoregressive models (AR), assume that X_t is a linear function of previous values.[4]

$$X_t = \sum_{i=1}^{p} \phi_i X_{t-i} + \varepsilon_t \qquad (1)$$

Where ϕ_i , in this equation are the coefficients of auto-regression.

The autocorrelation function is a very important function in the characterization of a linear process, it’s written as follow [2]:

$$\rho(h) = \frac{\gamma(h)}{\gamma(0)} = \frac{\sum_{i=1}^{N}(X_i - \overline{X})(X_{i+h} - \overline{X})}{\sum_{i=1}^{N}(X_i - \overline{X})^2} \qquad (2)$$

Where $\gamma(h)$ is the auto-covariance function and $\overline{X}$ is the average.

This function provides information about the process memory, that is to say, the degree of dependence between observations at time *t* and those made at the time *t-h* .

2.2. The Autoregressive process coefficients

For P = 1, it is easy to see that $\phi_1 = \gamma(1)$ and $\sigma^2 = 1 - \phi_{11}^2$, for higher orders $(p \geq 2)$, the coefficients of the AR process are calculated using the following recurrence relations : [2]

$$\phi_{hh} = \frac{\left[\gamma(h) - \sum_{p=1}^{h-1} \phi_{h-1,p}(h-p)\right]}{\sigma_{h-1}^2} \quad (3)$$

$$\phi_{h,p} = \phi_{h-1,p} - \phi_{hh}\phi_{h-1,h-p} \quad (4)$$

$$\sigma_h^2 = 1 - \sum_{i=1}^{h} \phi_{hi}^2 \quad (5)$$

If the process is of the order p , then:

$$\sigma_{p+1}^2 = \sigma_p^2$$

$$\phi_{p+1,i} = \phi_{p,i}$$

$$\phi_{p+1,p+1} = 0$$

The order of an autoregressive process is the value for which the partial autocorrelation function is zero. Estimated partial autocorrelation coefficients are considered invalid if they fall within the confidence interval constructed from the standard deviations. It is recognized that a series follow an AR (p) if the correlogram of its autocorrelation function decreases to 0 and its partial autocorrelation function vanishes beyond a lag p.

3. Database

To build our database, we considered three months of satellite observation in the Mediterranean area (the channel of Meteosat IR03.9) (see Fig.1.a and Fig.1.b), ie November and December 2009 and January for the 2010 year. For each day we recorded 24 images, the pixels of those images are coded over 8 bits and with a spatial resolution of 5kmx5km.

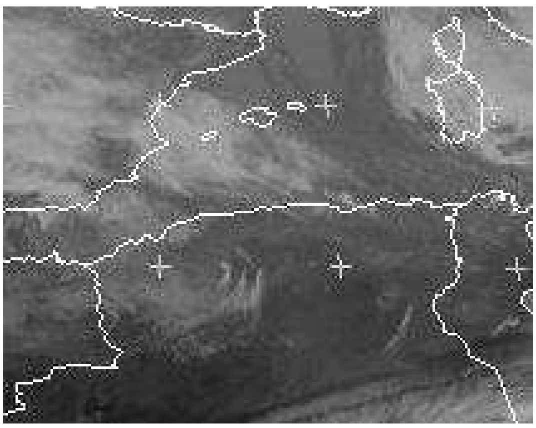

Figure 1.a. Satellite outlined image of the Mediterranean area

Figure 1.b. Satellite image of the Mediterranean area (without outline)

When we treated the satellite images of the Mediterranean region we have seen that the echoes of rain on these images were characterized by high persistence. In addition, we found that the echoes of rain were standing over a period of more than a month.
Such properties imply that changes in precipitation over time can be described by a sequence of random variables forming an autoregressive process.

4. Data processing

In this study, the Satellite images are divided into two zones; one corresponds to the sea, the other one corresponds to the continent (see Fig.2). For a detailed analysis of the phenomenon, we divided our work on two parts: processing pixel by pixel and processing window by window of size 5x5 pixels. Left and right shifts in both zones are applied to study a good part of our image, which has a format of 361 * 511 pixels.

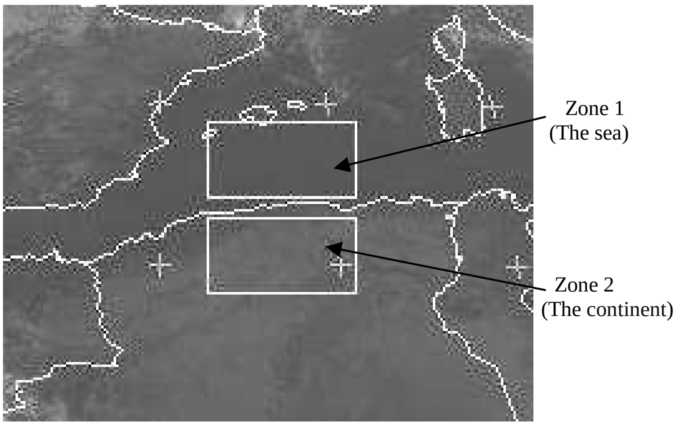

Figure 2. Image shows the two study zones

The precipitations versus time for both areas are represented by figures (3.a and 3.b). These curves are respectively obtained for process by pixels (average of 1000 pixels) and process by windows (5x5 pixels) (average of 40 windows) for a period of 91 days.

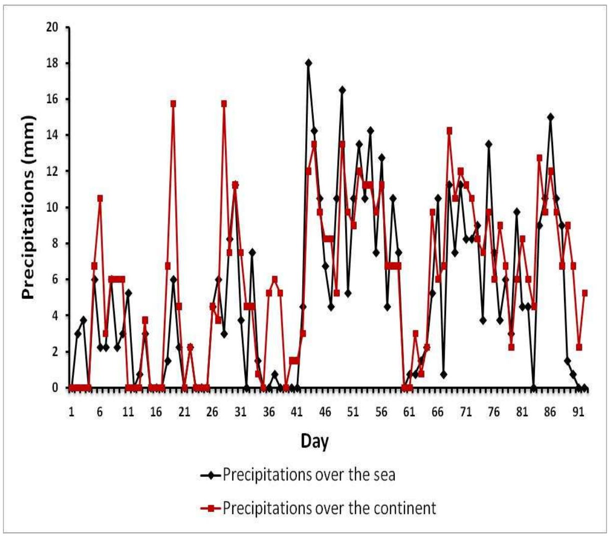

Figure 3.a. Daily precipitations over both sea (Zone 1) and continent(Zone 2) (processing by pixels)

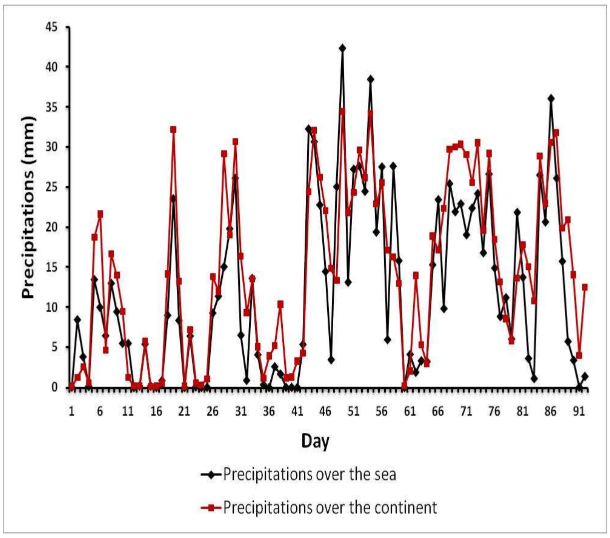

Figure 3.b. Daily precipitations over both sea (Zone 1) and continent(Zone 2) (processing by windows)

From the previous curves, it is clear that the data series are stationary as they exhibit fluctuations in rainfall, around an average value, 5.55 for the pixel by pixel process and 13.27 for window by window process.

5. Results and interpretations

The following figures show the variation of the first 15 autocorrelation coefficients (AC) and partial autocorrelation coefficients (PAC) for both areas.
The confidence band is given inside the interval [-0.2, 0.2].

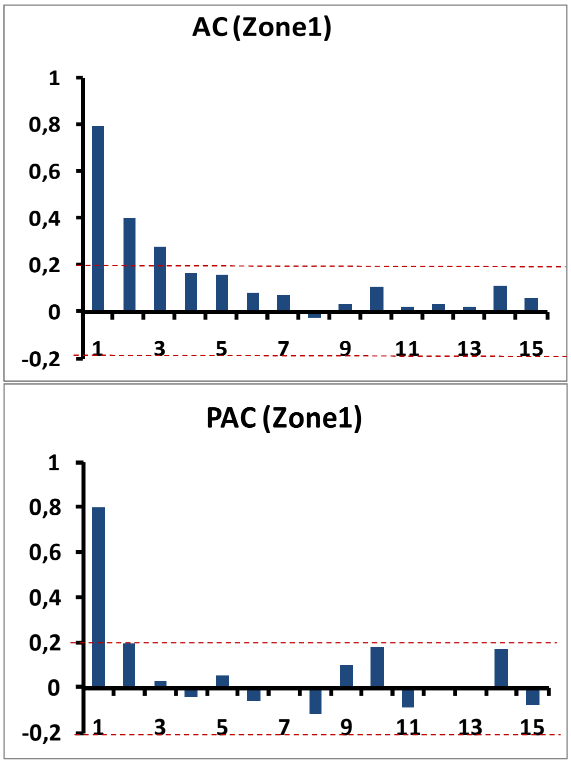

Figure 4.a. Variations of autocorrelation coefficients (AC) and partial autocorrelation (PAC).
(Processing by pixels for zone 1)

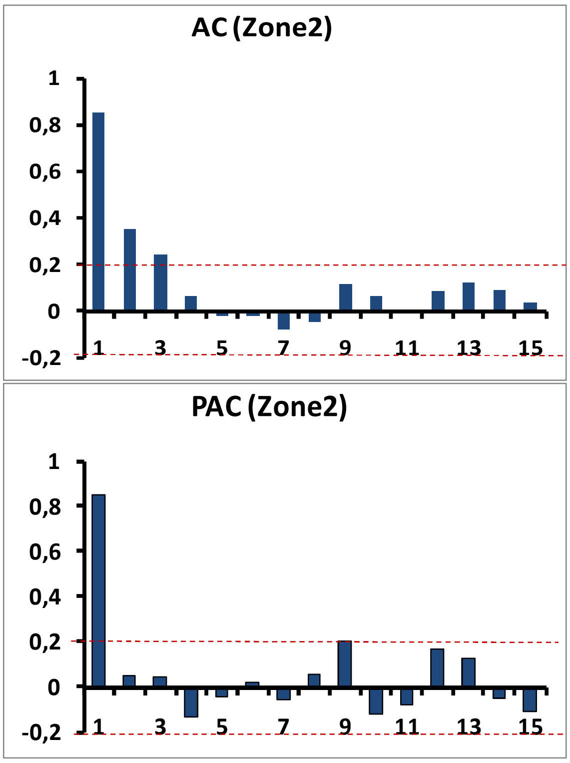

Figure 4.b. Variations of autocorrelation coefficients (AC) and partial autocorrelation (PAC).
(Processing by pixels for zone 2)

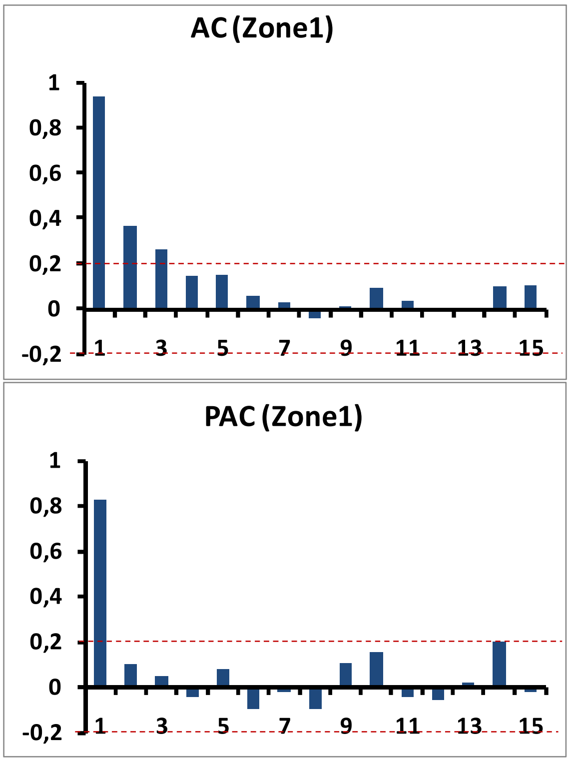

Figure 5.a. Variations of autocorrelation coefficients (AC) and partial autocorrelation (AC).
(Processing by windows for zone 1)

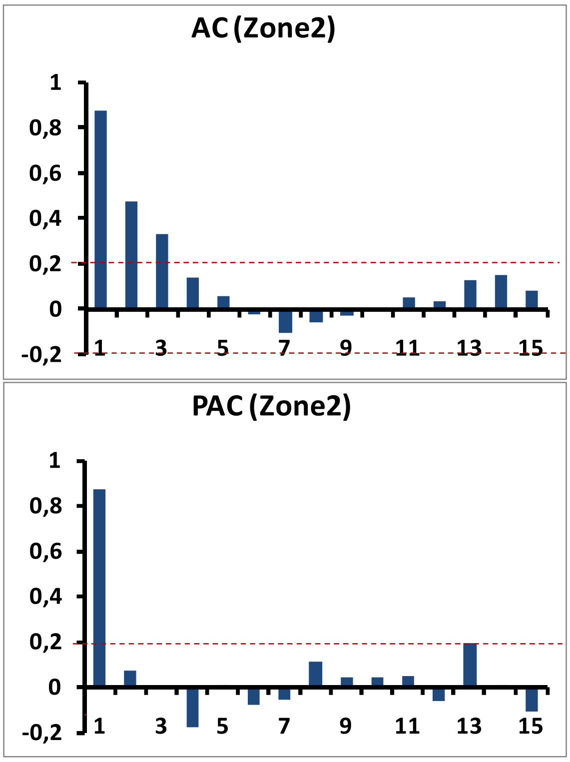

Figure 5.b. Variations of autocorrelation coefficients (AC) and partial autocorrelation (AC).
(Processing by windows for zone 2)

We note that the autocorrelation coefficients decrease exponentially on the one hand and the partial autocorrelation coefficients are within the confidence band defined by the interval [-0.2, 0.2].

For the values P=1, 2 and 3, the coefficients of the AR process are calculated and presented in the following table:

	Zone 1	**Zone2**
Processing by pixels	AR(1): ϕ_{11}= 0.8000 σ_1^2= 0.3600 AR(2): ϕ_{21}= 0.6456 ϕ_{22}= 0.1930 σ_2^2= 0.5460 AR(3): ϕ_{31}= 0.6403 ϕ_{32}= 0.1753 ϕ_{33}= 0.0274 σ_3^2= 0.5585	AR(1): ϕ_{11}= 0.8502 σ_1^2= 0.2772 AR(2): ϕ_{21}= 0.8080 ϕ_{22}= 0.0496 σ_2^2= 0.3446 AR(3): ϕ_{31}= 0.8060 ϕ_{32}= 0.0160 ϕ_{33}= 0.0416 σ_3^2= 0.3484
Processing by windows	AR(1): ϕ_{11}= 0.8235 σ_1^2= 0.3218 AR(2): ϕ_{21}= 0.7398 ϕ_{22}= 0.1017 σ_2^2= 0.4424 AR(3): ϕ_{31}= 0.7350 ϕ_{32}= 0.0674 ϕ_{33}= 0.0464 σ_3^2= 0.4530	AR(1): ϕ_{11}= 0.8710 σ_1^2= 0.2414 AR(2): ϕ_{21}= 0.8099 ϕ_{22}= 0.0701 σ_2^2= 0.3391 AR(3): ϕ_{31}= 0.8104 ϕ_{32}= 0.0755 ϕ_{33}= -0.0067 σ_3^2= 0.3375

Table 1. The coefficients of the AR process.

The AR model was applied to all pixels and windows, where the parameter ϕ_{ij} varies between 0 and 0.8710. The analysis of the partial autocorrelation coefficients ϕ_{ij} shows that:

- The process is stationary to the first order.
- The coefficients ϕ_{ip} of the zone continent are slightly higher than the coefficients of the zone sea.
- The variance of the error is larger over sea than over continent.
- The AR(1) is well suited to describe precipitation.

6. Validation

After identifying the model, it is important to verify the adequacy of the model used with the observations. As the errors are not observable, they are replaced by the estimated errors calculated from the estimated parameters of model:

$$\hat{\varepsilon}_t = X_t - \sum_{i=1}^{p} \phi_{pi} X_{t-i} \quad (6)$$

The fit test will be based on the chi-square test and the autocorrelation function of the estimated errors. This function is written as:

$$\rho_h\left(\hat{\varepsilon}_t\right) = \frac{\sum_{t=1}^{n-h}\left(\hat{\varepsilon}_t\right)\left(\hat{\varepsilon}_{t+h}\right)}{\sum_{t=1}^{n}\hat{\varepsilon}_t^{\,2}} \quad (7)$$

However, because of some dispersion in the calculation of autocorrelation coefficients, it is better to consider them in their entirety. Thus, we calculate the first 15 autocorrelation errors.

We then use the statistical distribution given by Box and Pierce [2]:

$$Q_n = n\sum_{h=1}^{15}\rho_h^2\left(\hat{\varepsilon}_t\right) \quad (8)$$

Where n is the total number of samples studied.

We compare the values of Q_n with the threshold χ^2 read on the table of chi-square, which is 23.68 for 14(15-P(=1)) degrees of freedom and a risk of error of α=0.05.

Q_n	Zone 1	Zone 2
Process by pixels	20.53	18.32
Process by windows	14.53	14.66

Table 2. Values of Q_n for both areas

The results given in Table 2. show that Q_n is still below the threshold χ^2 for both areas (sea and land). The errors are independent of each other.

7. Conclusion

The results of the statistical analysis of images showed that the rainfall is well described by an autoregressive process of order one (AR (1)), irrespective of their types, over sea or land. This result is in good agreement with those published in the literature. This work showed that the historical rainfall data is limited only to the previous day.

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9. References

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