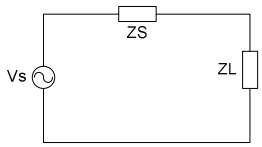
Maximum Power Transfer

Consider the following circuit: The input voltage is sinusoidal with a peak value V_s. The source impedance is $Z_S = R_S + jX_S$ and the load impedance is $Z_L = R_L + jX_L$.



The magnitude of the current flowing in the circuit is:

$$I = \frac{V_s}{|Z_S + Z_L|}$$

The power dissipated by the load is:

$$P_L = I_{rms}^2 R_L = \frac{V_s^2 R_L}{2|Z_S + Z_L|^2} = \frac{V_s^2 R_L}{2\left[\left(R_S + R_L\right)^2 + \left(X_S + X_L\right)^2\right]}$$

If we want to find the load that maximizes the above power, we have to solve the equations:

$\nabla P_L = 0$ or the system of equations: $\begin{cases} \dfrac{\partial P_L}{\partial R_L} = 0 \\ \dfrac{\partial P_L}{\partial X_L} = 0 \end{cases}$

and $\dfrac{\partial P_L}{\partial X_L} = \dfrac{V_s^2 R_L \left(X_S + X_L\right)}{\left[\left(X_S + X_L\right)^2 + \left(R_S + R_L\right)^2\right]^2}$

$$\frac{\partial P_L}{\partial R_L} = \frac{V_s^2\left[\left(X_S + X_L\right)^2 + \left(R_S + R_L\right)^2 - 2R_L\left(R_S + R_L\right)\right]}{2\left[\left(X_S + X_L\right)^2 + \left(R_S + R_L\right)^2\right]^2}$$

Setting both equations to zero provides: $X_L = -X_S$ and $R_L = R_S$. In other words: $Z_L = Z_S^*$.

Conclusion:

In order to transfer the maximum of power from a source, the load must be matched: The load reactance should cancel the source reactance and the load resistance must be equal to the source one.