Numerical Methods Control Test

Ex1 (7 points):

Find the root of f(x) = $x^2 - 3$ using Bisection method with an error of $e \cong 0.01$

Ex2 (6 points):

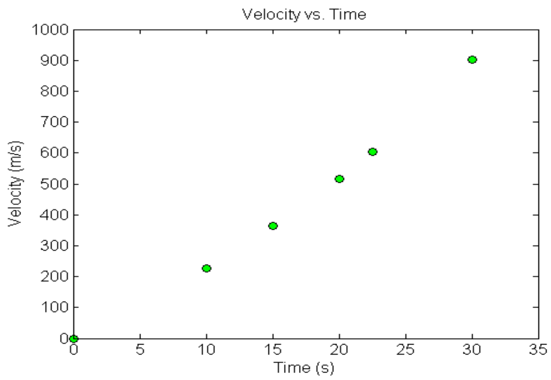
Use the formula of Newton-Raphson method (first order) to solve:

$$f(x) = x\log_{10}(x) - 2 = 0$$

Ex3 (7 points):

The upward velocity of a rocket is given as a function of time in the table below.

t (s)	$v(t)$ (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67



Graph of velocity vs. time data for the rocket

Determine the value of the velocity at $t = 16$ seconds.

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Solution:

Ex1:(7 points)

The initial interval [1 2]

Left Endpoint	Right Endpoint	Midpoint	P(Midpoint)	
1.0	2.0	1.5	-0.75	**(1)**
1.5	2.0	1.75	0.062	**(1)**
1.5	1.75	1.625	-0.359	**(1)**
1.625	1.75	1.6875	-0.1523	**(1)**
1.6875	1.75	1.7188	-0.0457	**(1)**
1.7188	1.75	1.7344	0.0081	**(1)**

Thus, with the six iterations, the final interval, [1.7188, 1.7344], has an error $e \cong 0.01$. Therefore we take as an approximation of the root = 1.7344 **(1)**.

Ex2: (6 points) The formula of Newton-Raphson method is: $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$ **(1)**

We have $f(3) = -0.6$ and $f(4) = +0.4$, whence we use $x_0 = 3.5$ as a first approximation. **(1)**

Then, we obtain the successive approximations:

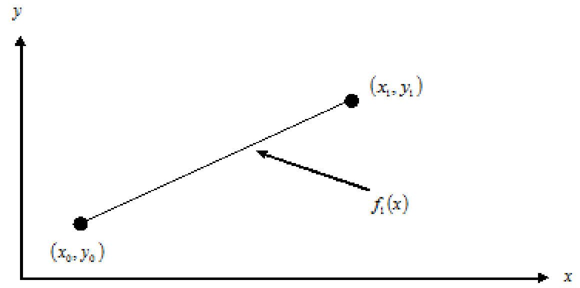
$x_0 = 3.5$; $x_1 = 3.598$; $x_2 = 3.597284$; $x_3 = 3.597285$

(1p) **(1p)** **(1p)** **(1p)**

Ex 3): (7 points)

For the first order polynomial interpolation (also called linear interpolation), the velocity is given by:

$$v(t) = \sum_{i=0}^{1} L_i(t)v(t_i) = L_0(t)v(t_0) + L_1(t)v(t_1) \quad \textbf{(1p)}$$



Since we want to find the velocity at $t = 16$, and the graph represents a linear regression, we need to choose the two data points closest to $t = 16$ are $t_0 = 15$ and $t_1 = 20$. **(1p)**

Then: $t_0 = 15,\ v(t_0) = 362.78$ and $L_0(t) = \prod_{\substack{j=0 \\ j\neq 0}}^{1} \dfrac{t - t_j}{t_0 - t_j} = \dfrac{t - t_1}{t_0 - t_1}$ **(1p)**

$t_1 = 20,\ v(t_1) = 517.35$ and $L_1(t) = \prod_{\substack{j=0 \\ j\neq 1}}^{1} \dfrac{t - t_j}{t_1 - t_j} = \dfrac{t - t_0}{t_1 - t_0}$ **(1p)**

Hence $v(t) = \dfrac{t - t_1}{t_0 - t_1} v(t_0) + \dfrac{t - t_0}{t_1 - t_0} v(t_1) = \dfrac{t - 20}{15 - 20}(362.78) + \dfrac{t - 15}{20 - 15}(517.35), \quad 15 \le t \le 20$

(1p) **(1p)**

$$v(16) = \frac{16 - 20}{15 - 20}(362.78) + \frac{16 - 15}{20 - 15}(517.35) = 0.8(362.78) + 0.2(517.35) = 393.69 \text{ m/s} \quad \textbf{(1p)}$$

Dr. BOUSHAKI

DGEE/Promotion 05 **Tuesday,December 14,2004**

Numerical Methods
First Mid-term Exam

Ex.1:(10pts)

Construct the flow chart of polynomial method and its program with fortran 90.

Ex.2: (10pts)

a) Find the solution of linear system using Gauss-Seidel method:

$$\begin{cases} 10x_1 + x_2 + 2x_3 = 44 \\ 2x_1 + 10x_2 + x_3 = 51 \\ x_1 + 2x_2 + 10x_3 = 61 \end{cases}$$

b) Given the function $f(x) = Tang(40x)$. Find $f'(0.175)$ using backward difference representation of $O(h)$ with h=0.075.

c) Find the inversion of the matrix:

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

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Numerical Methods Exam

Ex1 (5 points):

x_i	3	4.5	7	9
$f(x_i)$	2.5	1	2.5	0.5

We assume that, the first derivatives are continuous and the 2nd derivative is zero at the first point.
Using the given data points in different intervals, interpolate by spline method the function of this system: $Q_i(x)=a_i x^2+b_i x+c_i$

Ex2 (5 points):

n	0	1	1.5	2
x	-1	0	1	2
y	6	-2	-4	12

Interpolate to find $P_3(x)$.

Ex3 (5 points):

$$A = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$$

Estimate the dominant eigenvector $v = \begin{bmatrix} x \\ y \end{bmatrix}$ by the power method.

Ex4 (5 points):

Develop a program to solve this equation with small error.

$\mathbf{y' = x + y}$, **with** $\mathbf{y(0) = y_0}$

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Solution

Ex1:(5 points)

We take three intervals: [3, 4.5] , [4.5 , 7] and [7, 9]

Satisfying the three cubic spline conditions:

1. Interpolating conditions

$$\begin{aligned}
9a_1 &+ 3b_1 &+ c_1 &= 2.5\\
20.25a_1 &+ 4.5b_1 &+ c_1 &= 1.0\\
20.25a_2 &+ 4.5b_2 &+ c_2 &= 1.0\\
49a_2 &+ 7b_2 &+ c_2 &= 2.5\\
49a_3 &+ 7b_3 &+ c_3 &= 2.5\\
81a_3 &+ 9b_3 &+ c_3 &= 0.5
\end{aligned}$$

2. Continuous first derivatives

$$\begin{aligned}
9a_1 + b_1 &= 9a_2 + b_2\\
14a_2 + b_2 &= 14a_3 + b_3
\end{aligned}$$

3. Assume the 2nd derivatives is zero at the first point.

$$a_1 = 0$$

We can write the system of equations in matrix form as

$$\begin{bmatrix}
9 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
20.25 & 4.5 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 20.25 & 4.5 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 49 & 7 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 49 & 7 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 81 & 9 & 1\\
9 & 1 & 0 & -9 & -1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 14 & 1 & 0 & -14 & -1 & 0\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} a_1\\ b_1\\ c_1\\ a_2\\ b_2\\ c_2\\ a_3\\ b_3\\ c_3 \end{bmatrix}
=
\begin{bmatrix} 2.5\\ 1\\ 1\\ 2.5\\ 2.5\\ 0.5\\ 0\\ 0\\ 0 \end{bmatrix}$$

The system of equations can be solved to yield

$$\begin{aligned}
a_1 &= 0 & b_1 &= -1 & c_1 &= 5.5\\
a_2 &= 0.64 & b_2 &= -6.76 & c_2 &= 18.46\\
a_3 &= -1.6 & b_3 &= 24.6 & c_3 &= -91.3
\end{aligned}$$

Thus the quadratic is: $Q(x) = \begin{cases} -x + 5.5 & x \in [3,4.5] \\ 0.64x^2 - 6.76x + 18.46 & x \in [4.5,7] \\ -1.6x^2 + 24.6x - 91.3 & x \in [7,9] \end{cases}$

Ex2: (5 points)

a) The time "n" to record data is not with the same bandwidth (time).

b) If we take in consideration (x,y):

x	-1	0	1	2
y	6	-2	-4	12

$$P_3(x) = f(x) = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} y_0 + \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} y_1$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2+\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$P_3(x)=f(x)=2x^3-x^2+x-2$$

c) We can calculate $P_2(x)$

$$P_2(x)=f(x)=\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0+\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1+\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}y_2$$

Putting the values, we get: $P_2(x)=2x(x-2)+(x+1)(x-2)+2x(x+1)$; $P_2(x)=5x^2-3x-2$

Ex 3: (5 points)

Assume the initial dominant eigenvector $x_0=\begin{bmatrix}3\\1\end{bmatrix}$

$$x_1=Ax_0=\begin{bmatrix}2 & 4\\1 & -1\end{bmatrix}\begin{bmatrix}3\\1\end{bmatrix}=\begin{bmatrix}10\\2\end{bmatrix}=2\begin{bmatrix}5\\1\end{bmatrix}$$

$$x_2=Ax_1=A^2x_0\begin{bmatrix}2 & 4\\1 & -1\end{bmatrix}\begin{bmatrix}10\\2\end{bmatrix}=\begin{bmatrix}28\\8\end{bmatrix}=8\begin{bmatrix}3.5000\\1.0000\end{bmatrix}$$

$$x_3=Ax_2=A^3x_0=\begin{bmatrix}2 & 4\\1 & -1\end{bmatrix}\begin{bmatrix}28\\8\end{bmatrix}=\begin{bmatrix}88\\20\end{bmatrix}=20\begin{bmatrix}4.4000\\1.0000\end{bmatrix}$$

$$x_4=Ax_3=A^4x_0=\begin{bmatrix}2 & 4\\1 & -1\end{bmatrix}\begin{bmatrix}88\\20\end{bmatrix}=\begin{bmatrix}256\\68\end{bmatrix}=68\begin{bmatrix}3.7647\\1.0000\end{bmatrix}$$

$$x_5=Ax_4=\begin{bmatrix}2 & 4\\1 & -1\end{bmatrix}\begin{bmatrix}256\\68\end{bmatrix}=\begin{bmatrix}784\\188\end{bmatrix}=188\begin{bmatrix}4.1702\\1.0000\end{bmatrix}$$

$$x_6=Ax_5=\begin{bmatrix}2 & 4\\1 & -1\end{bmatrix}\begin{bmatrix}784\\188\end{bmatrix}=\begin{bmatrix}2320\\596\end{bmatrix}=596\begin{bmatrix}3.8926\\1.0000\end{bmatrix}$$

$$x_7=Ax_6=\begin{bmatrix}2 & 4\\1 & -1\end{bmatrix}\begin{bmatrix}2320\\596\end{bmatrix}=\begin{bmatrix}7024\\1724\end{bmatrix}=1724\begin{bmatrix}4.0742\\1.0000\end{bmatrix}$$

$$x_8=Ax_7=\begin{bmatrix}2 & 4\\1 & -1\end{bmatrix}\begin{bmatrix}7024\\1724\end{bmatrix}=\begin{bmatrix}20944\\5300\end{bmatrix}=5300\begin{bmatrix}3.9517\\1.0000\end{bmatrix}$$

$$x_9=Ax_8=\begin{bmatrix}2 & 4\\1 & -1\end{bmatrix}\begin{bmatrix}20944\\5300\end{bmatrix}=\begin{bmatrix}63088\\15644\end{bmatrix}=15644\begin{bmatrix}4.0327\\1.0000\end{bmatrix}$$

We see that approximation turne about the dominant eigenvector $v=\begin{bmatrix}4\\1\end{bmatrix}$ so we can take $x=\begin{bmatrix}4.1702\\1.0000\end{bmatrix}$ as approximate dominant eigenvector.

Ex 4: (5 points)

a) Program of Euler :

$$y_{n+1}=y_n+hf(x_n,y_n),\qquad n=0,1,2,\ldots\ldots,N-1$$

b) Program of Runge Kutta first order:

$$y(x_1)\approx y_1=y(x_0)+hy'(x_0)+\frac{h^2}{2!}y''(x_0)$$

c) Program of Runge Kutta second order:

$K_1=hf(x_n,y_n)$

$K_2=hf(x_n+h,\ y_n+k_1)$

$y_{n+1}=y_n+\frac{1}{2}(K_1+K_2)$

With : $x_{i+1}=x_i+h$

Dr. BOUSHAKI

DGEE/Promotion EO5 **Sunday,Dec 11,2005**

Numerical Methods
First Mid-term Exam

Ex1

Solve the following system by Gauss elimination:

$5.6 x + 3.8 y + 1.2 z = 1.4$

$3.l x + 7.l y - 4.7 z = 5.1$

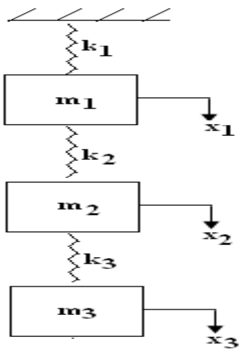
$1.4 x - 3.4y + 8.3 z = 2.4$

Ex2

Determine the root(roots) by Newton second order of the equation : $2 \cos(x) - e^x = 0$

Ex3

Analyze the free undamped vibrational characteristics of the three degrees – of– freedom system masses m_1,m_2 and m_3 connected by the three springs, with spring constants k_1,k_2 and k_3.



$m_1=m_2=m_3=14.59$kg $k_1=k_2=k_3=145.9$N/m(Newton /meter)

The displacements of the masses are defined by the generalized coordinates x_1,x_2 and x_3 respectively, each displacement being measured from the static-equilibrium position of the respective mass.

Find the three equations of the motion of the system

2) Set: $x_1=X_1 \sin(pt)$, $x_2=X_2 \sin(pt)$, $x_3=X_3 \sin(pt)$

Where X_1,X_2 and X_3 are the amplitudes of the motion of the respective masses, and " p" denotes the natural circular frequencies.

What is(are) the value(s) of " p" for what the system of algebraic equations has an infinite number of solutions.

Ex4

Derive the Newton-Raphson iteration formula: $x_{n+1} = x_n - (x^k_n - a)/k\, x^{k-1}_n$

for finding the k-th root of a.

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Solution :

Ex1:

m	*Augmented Matrix*				*Check*
	5·6	3·8	1·2	1·4	12·0
	3·1	7·1	−4·7	5·1	10·6
	1·4	−3·4	8·3	2·4	8·7
	5·6	3·8	1·2	1·4	12·0
−0·554		4·99	−5·36	4·32	3·95
−0·250		−4·35	8·00	2·05	5·70
	5·6	3·8	1·2	1·4	12·0
		4·99	−5·36	4·32	3·95
+0·872			3·33	5·82	9·14 (9·15)

Working to 2*D* (rounded)

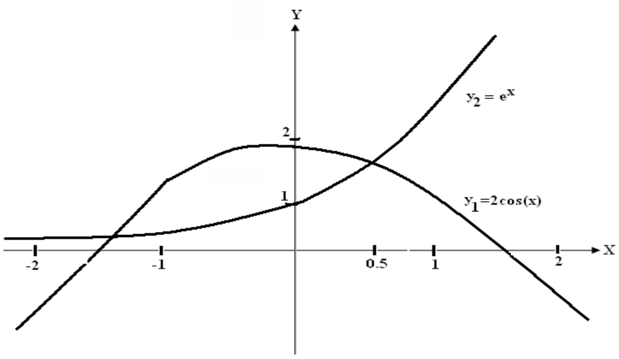
Solution by back-substitution

$$\left.\begin{aligned} 3{\cdot}33z &= 5{\cdot}83 \rightarrow z = 1{\cdot}75 \\ 4{\cdot}99y - 5{\cdot}36 \times 1{\cdot}75 &= 4{\cdot}32 \rightarrow y = 2{\cdot}75 \\ 5{\cdot}6x + 3{\cdot}8 \times 2{\cdot}75 + 1{\cdot}2 \times 1{\cdot}75 &= 1{\cdot}4 \rightarrow x = -1{\cdot}99 \end{aligned}\right\}$$

Residuals

$$1{\cdot}4 - (-11{\cdot}14 + 10{\cdot}45 + 2{\cdot}10) = -0{\cdot}01$$
$$5{\cdot}1 - (-6{\cdot}17 + 19{\cdot}53 - 8{\cdot}23) = -0{\cdot}03$$
$$2{\cdot}4 - (-2{\cdot}79 - 9{\cdot}35 + 14{\cdot}53) = 0{\cdot}01$$

Ex2



x	F(x)= 2 cos(x)-e^x	F'(x)=-2sin(x)-e^x	F''(x)=-2cos(x)-e^x	$\left[\dfrac{f(x_n)}{f'(x_n) - \left(\dfrac{f''(x_n).f(x_n)}{2.f'(x_n)}\right)}\right]$
X_n=0.4	0.350	-2.270	-3.334	-0.139
X_{n+1}=0.539	0.003	-2.741	-3.433	-0.001
X_{n+2}=0.540	0.000			

Ex3: The differential equations are:

$$
\begin{aligned}
&m_1 \ddot{x}_1 + (k_1 + k_2)x_1 - k_2 x_2 = 0 \\
&m_2 \ddot{x}_2 - k_2 x_1 + (k_2 + k_3)x_2 - k_3 x_3 = 0 \qquad (1)\\
&m_3 \ddot{x}_3 - k_3 x_2 + k_3 x_3 = 0
\end{aligned}
$$

Take:

$$
\begin{aligned}
&x_1 = X_1 \sin(pt) \\
&x_2 = X_2 \sin(pt) \qquad (2)\\
&x_3 = X_3 \sin(pt)
\end{aligned}
$$

We replace (2) in (1) we get:

$$
\begin{aligned}
&(20 - p^2)X_1 - 10X_2 = 0 \\
&-10X_1 + (20 - p^2)X_2 - 10X_3 = 0 \qquad (3)\\
&-10X_2 + (10 - p^2)X_3 = 0
\end{aligned}
$$

To obtain an infinite number of solutions the determinant of the coefficients of Xi must be equal to zero:

$$
\begin{vmatrix}
(20 - p^2) & -10 & 0 \\
-10 & (20 - p^2) & -10 \\
0 & -10 & (10 - p^2)
\end{vmatrix} = 0 \qquad (4)
$$

Expansion of this determinant results in: $p^6 - 50p^4 + 600p^2 - 1000 = 0$

Which may be written as : $(p^2)^3 - 50(p^2)^2 + 600(p^2) - 1000 = 0$

The roots are: $p_1^2 = 1.98s^{-2}$, $p_2^2 = 15.5s^{-2}$, $p_3^2 = 32.5s^{-2}$

Ex4:

$$x^k = a$$

$$f(x) = x^k - a = 0$$

$$f'(x) = kx^{k-1}$$

$$x_{n+1} = x_n - \frac{x_n{}^k - a}{kx_n{}^{k-1}}$$

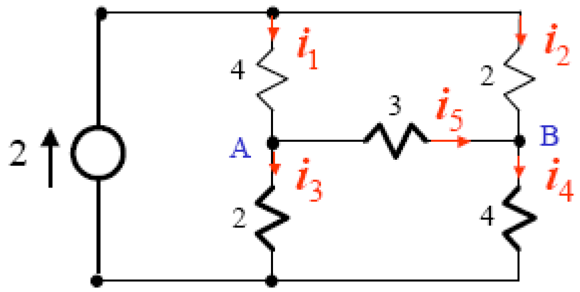
Mrs BOUSHAKI

Boumerdes university Sunday, 24 december 2006

Faculty of Engineering Sciences

Numerical Methods First Exam

Ex. 1



Use the Gauss elimination to determine the currents i_1 , i_2 , i_3 , i_4 *and* i_5

Ex. 2

a) Find $\sqrt{8}$ correct to within 4 decimal places.

b) Suppose we wish to construct a polynomial P_5 that interpolates a function $f \in [-1,+1]$ such as: $P_5(-1) = f(-1)$; $P_5'(-1) = f'(-1)$; $P_5(0) = f(0)$; $P_5''(0) = f''(0)$; $P_5(1) = f(1)$; $P_5'(1) = f'(1)$. Write the linear system to determine the coefficients $c_0, \dots, c_5$ for the polynomial: $p_5(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5$.

Ex. 3

a)

Show that the equation has a root: $x^2 + e^x - 5 = 0$

b)

We wish to see if two groups of nurses distribute their time in six different categories about the same way. That is, we wish to test H: if $p_{i1} = p_{i2}$ for $i = 1, \dots, 4$. To carry out this test, nurses were observed at random throughout several days, each observation resulting in a mark in one of the six categories. The data are summarised below.

Category	1	2	3	4	Total
Group I	90	30	70	20	300
Group II	50	20	40	10	200

Perform the test and state your conclusions.

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Solution

Ex.1

KCL at Node A: $i_1 - i_3 - i_5 = 0$

KCL at Node B: $i_2 - i_4 + i_5 = 0$

KVL to left loop: $4i_1 + 2i_3 = 2$

KVL to right upper loop:

$$4i_1 - 2i_2 + 3i_5 = 0$$

KVL to right lower loop:

$$2i_3 - 4i_4 - 3i_5 = 0$$

In a matrix form:

$$\begin{bmatrix} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 \\ 4 & 0 & 2 & 0 & 0 \\ 4 & -2 & 0 & 0 & 3 \\ 0 & 0 & 2 & -4 & -3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

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Ex.2

a) Here we want to find the root of the equation $f(x) = x^2 - 8 = 0$. Let's start with $x_0 = 3$. Since $f'(x) = 2x$ then using Newton first order:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Iterations(n)	x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}
0	3	1	6	2.8333
1	2.8333	0.0275	5.6666	2.8284
2	2.8284	-0.0001	5.6568	2.8284

The root is x=2.8284

b) We seek the coefficients $c_0, \ldots, c_5$ to the polynomial

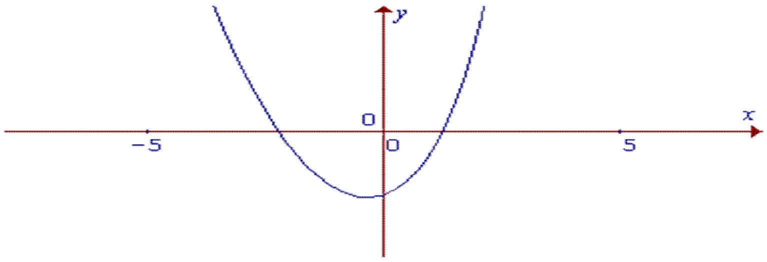
$$p_5(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5,$$

$$c_0 + c_1 x_0 + c_2 x_0^2 + c_3 x_0^3 + c_4 x_0^4 + c_5 x_0^5 = f(x_0)$$
$$c_1 + 2c_2 x_0 + 3c_3 x_0^2 + 4c_4 x_0^3 + 5c_5 x_0^4 = f'(x_0))$$
$$c_0 + c_1 x_1 + c_2 x_1^2 + c_3 x_1^3 + c_4 x_1^4 + c_5 x_1^5 = f(x_1)$$
$$2c_2 + 6c_3 x_0 + 12c_4 x_0^2 + 20c_5 x_0^3 = f''(x_1)$$
$$c_0 + c_1 x_2 + c_2 x_2^2 + c_3 x_2^3 + c_4 x_2^4 + c_5 x_2^5 = f(x_2)$$
$$c_1 + 2c_2 x_2 + 3c_3 x_2^2 + 4c_4 x_2^3 + 5c_5 x_2^4 = f'(x_2).$$

Ex.3

a)

Below is a graph of $y = x^2 + e^x - 5$



You can see that it crosses the axes around x = -2 and x = 1

Numerical Methods
First Mid-term Exam

Ex1

a) Explain the General treatment of the Gauss elimination process

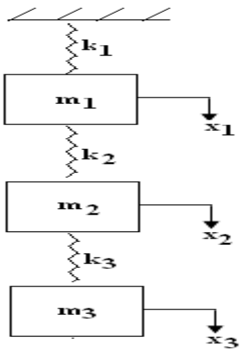
b) Explain the steps for finding the $f^{"}{'}x)+\theta(h)^{2}$

Ex2 :

Proof the Newton second order : $$x_{n+1} = x_n - \left[\frac{f(x_n)}{f'(x_n) - \left(\frac{f''(x_n).f(x_n)}{2.f'(x_n)} \right)} \right]$$

Ex3

Analyze the free undamped vibrational characteristics of the three degrees – of– freedom system masses m_1,m_2 and m_3 connected by the three springs, with spring constants k_1,k_2 and k_3.



$m_1=m_2=m_3=14.59$kg $k_1=k_2=k_3=145.9$N/m(Newton /meter)

The displacements of the masses are defined by the generalized coordinates x_1,x_2 and x_3 respectively, each displacement being measured from the static-equilibrium position of the respective mass.

find the three equations of the motion of the system

2) Set: $x_1=X_1 \sin(pt)$, $x_2=X_2 \sin(pt)$, $x_3=X_3 \sin(pt)$

Where X_1,X_2 and X_3 are the amplitudes of the motion of the respective masses, and " p" denotes the natural circular frequencies. Write the linear system : A.x=b

Ex4:

a) find the k-th root of a: $x_{n+1} = x_n - (x^k_n - a)/k\, x^{k-1}_n$

b) Explain the polynomial method to solve a linear system: A.x=b

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Solution :

Ex1:

Ex2

Ex3:

The differential equations are:

$$
\begin{aligned}
&m_1 \ddot{x}_1 + (k_1 + k_2)x_1 - k_2 x_2 = 0 \\
&m_2 \ddot{x}_2 - k_2 x_1 + (k_2 + k_3)x_2 - k_3 x_3 = 0 \qquad (1) \\
&m_3 \ddot{x}_3 - k_3 x_2 + k_3 x_3 = 0
\end{aligned}
$$

Take:

$$
\begin{aligned}
&x_1 = X_1 \sin(pt) \\
&x_2 = X_2 \sin(pt) \qquad (2) \\
&x_3 = X_3 \sin(pt)
\end{aligned}
$$

We replace (2) in (1) we get:

$$
\begin{aligned}
&(20 - p^2) X_1 - 10 X_2 = 0 \\
&-10 X_1 + (20 - p^2) X_2 - 10 X_3 = 0 \qquad (3) \\
&-10 X_2 + (10 - p^2) X_3 = 0
\end{aligned}
$$

Ex4:

$$x^k = a$$

$$f(x) = x^k - a = 0$$

$$f'(x) = kx^{k-1}$$

$$x_{n+1} = x_n - \frac{x_n^{\,k} - a}{kx_n^{\,k-1}}$$

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DGEE/Promotion EO5 **Saturday,29/11/2008**

Numerical Methods
First Exam

Ex1

a) What are pivot elements? Why must small pivot elements be avoided if possible
b) What is the **convergence criterion** for the bisection method?

Ex2 :

Find the inverses of the following matrix, using **elimination and back-substitution**:

$$A = \begin{bmatrix} 0.20 & 0.24 & 0.12 \\ 0.10 & 0.24 & 0.24 \\ 0.05 & 0.30 & 0.49 \end{bmatrix}$$

Ex3

a) Give the program that reads the number of the month and tells the number of days in that month.
b) Correct this program

```
PROGRAM UsingBlocks(INPUT,OUTPUT);
CONST
Limit := 1000;
VAR
Amount =:REAL;
BEGIN
WRITE('Please enter the amount:');
READLN('Amount');
IF Amount =< Limit THEN
BEGIN
WRITELN('Your charge is accepted.');
WRITELN('Your price plus tax is
$',1.05*Amount:0:2)
{The semicolon is optional}
END;
IF Amount > Limit
BEGIN
WRITELN('The amount exceeds your credit limit.');
WRITELN('The maximum limit is $',Limit)
{The semicolon is optional}
END
WRITELN('Thank you for using Pascal credit card.');
WRITELN('Press ENTER to continue..');
READLN {The semicolon is optional}
END.
{ ------------------------------------------------------------ }
```

Ex4:

a) The number 2 and its powers are very important numbers in the computer field then give the program that gives the powers of two.
b) Correct this program

```
PROGRAM CaseOfWeights(INPUT,OUTPUT);
CONST
Quarter = 25;
Dime = 10;
Nickel = 5;
VAR
CoinWeight, Amount :real;
BEGIN
WRITE('Please enter the weight: ');
READLN(CoinWeight);
CASE CoinWeight OF
35 : Amount := Quarter;
7 : Amount := Dime;
15 : Amount := Nickel;
END;
WRITELN('The amount is ', Amount, ' cents.');
WRITELN('Press ENTER to continue..');
READLN
END.
{ ---------------------------------------------------------------------- }
```

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Solution :

Ex1:

a)To avoid division by zero and avoid error caused by division by small number

b) the convergence criterion is the $|x_{N+1} - X_N| \langle \varepsilon$

Ex2:

19	-34	12
-15.4	38.3	-15
7.5	-20	10

Ex 3)

a)

First way

```
CASE Month OF
1,3,5,7,8,10,12 : Days := 31;
4,6,9,11 : Days := 30;
2 :         BEGIN
                WRITE('Enter the year:');
                READLN(Year);
                IF YEAR MOD 4 = 0 THEN
                        Days :=29
                ELSE
                        Days :=28
            END;
```

Second way

```
PROGRAM DaysOfMonth1(INPUT,OUTPUT);
VAR
Days, Month, Year :INTEGER;
BEGIN
WRITE('Please enter the number of the month: ');
READLN(Month);
CASE Month OF
    1,3,5,7,8,10,12 : Days := 31;
    4,6,9,11 : Days := 30;
    2 :     BEGIN
              WRITE('Enter the year:');
               READLN(Year);
               IF YEAR MOD 4 = 0 THEN
                           Days :=29
                ELSE
                           Days :=28
            END;
END;
WRITELN('There are ',Days,' days in this month.');
WRITELN('Press ENTER to continue..');
READLN
END.
```

Result of run:

Run 1:

Please enter the number of the month: 2
Enter the year: 1987
There are 28 days in this month.

Press ENTER to continue..

Run 2:

Please enter the number of the month: 2
Enter the year: 1984
There are 29 days in this month.
Press ENTER to continue..

Run 3:

Please enter the number of the month: 12
There are 31 days in this month.
Press ENTER to continue..

b)

Limit = 1000; Amount =:REAL; READLN('Amount'); <= END	Limit = 1000; Amount :REAL; READLN(Amount); =< END;

Ex4:

a)

```
PROGRAM PowerTwo(INPUT, OUTPUT);
VAR
Base, Power, Start, Final :INTEGER;
BEGIN
    Base := 2;
    WRITE('Enter starting exponent:');
    READLN(Start);
    WRITE('Enter ending exponent:');
    READLN(Final);
    WRITELN;
    WRITELN('Number Power of two');
    FOR Power := Start TO Final DO
        BEGIN
            WRITE(Power:3);
            WRITELN(EXP(LN(Base)*Power):20:0)
        END;
    WRITELN;
    WRITELN('Press ENTER to continue..');
    READLN
END.
```

b)

CoinWeight, Amount :real;	CoinWeight : INTEGER; Amount: string

Numerical Methods
First Mid-term Exam

Ex1

Find a backward difference representation for $\frac{d^2 f}{dx^2}$ which is of $\theta(h^2)$.

Ex2

Consider the function $f(x) = \sin(10\pi.x)$. Find $f'(0)$ using forward difference representation of $\theta(h)$ and $\theta(h^2)$ with $h = 0.2$. Compare these results with each other and with the exact analytical answer.

Ex3

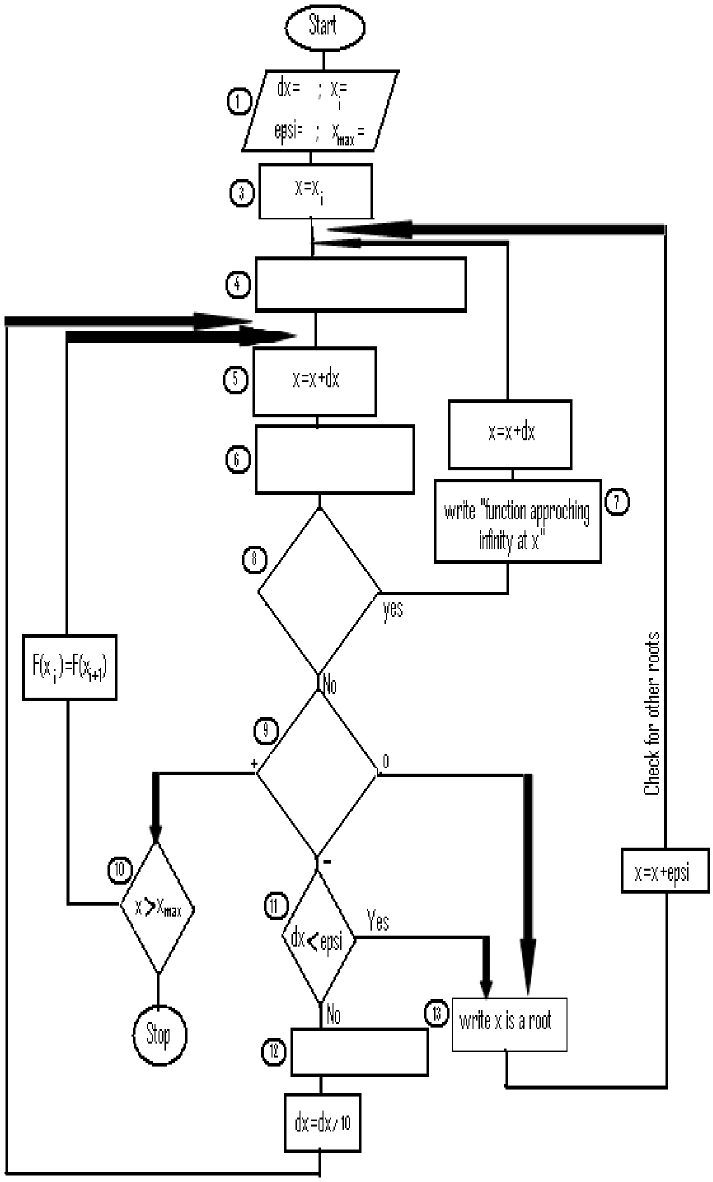
Use the Gauss-Seidel method to solve the following system of equations:

$$8x_1 + 2x_2 + 3x_3 = 30,$$
$$x_1 - 9x_2 + 2x_3 = 1,$$
$$2x_1 + 3x_2 + 6x_3 = 31.$$

Using initial condition $[x_1, x_2, x_3] = [1,1,1]$, take 4 digits after decimal point. And give the result in the table.

Ex4

Complete the flow chart



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Solution

Ex1

The Taylor series expansion for f(x) about the x=c is:

$$f_c(x) = f(x) + (x-c) f'(c) + \frac{(x-c)^2}{2!} f''(c) + \frac{(x-c)^3}{3!} f'''(c) + \frac{(x-c)^4}{4!} f^{(4)}(c) + \ldots\ldots\ldots\ldots(1)$$

And we have: $\nabla f_j = f_j - f_{j-1}$, $\nabla^2 f_j = \nabla(\nabla f_j) = \nabla f_j - \nabla f_{j-1} = f_j - 2 f_{j-1} + f_{j-2}$

$$\nabla^3 f_j = \nabla(\nabla^2 f_j) = \nabla f_j - 2\nabla f_{j-1} + \nabla f_{j-2} = f_j - 3 f_{j-1} + 3 f_{j-2} - f_{j-3}$$

So $f_j^{(3)} = \frac{\nabla^3 f_j}{h^3} = \frac{f_j - 3 f_{j-1} + 3 f_{j-2} - f_{j-3}}{h^3} + \theta(h) \ldots\ldots\ldots\ldots(2)$

Or we can write: $f^{(3)}(c) = f_c^{(3)} = \frac{\nabla^3 f_c}{h^3} = \frac{f(c) - 3f(c-h) + 3f(c-2h) - f(c-3h)}{h^3} + \theta(h) \ldots\ldots\ldots\ldots(2)$

$$(1) \Rightarrow f_c(x) = f(x) + (x-c) f'(c) + \frac{(x-c)^2}{2!} f''(c) + \frac{(x-c)^3}{3!} f'''(c) + \theta(h^2) \ldots\ldots\ldots\ldots\ldots(3)$$

$$At \quad x = c-h, \qquad (3) \Rightarrow f_c(c-h) = f(c) - hf'(c) + \frac{h^2}{2} f''(c) - \frac{h^3}{6} f^{(3)}(c) + \theta(h^2) \ldots\ldots\ldots\ldots\ldots(4)$$

$At \quad x = c-2h, \qquad (3) \Rightarrow f_c(c-2h) = f(c) - 2hf'(c) + 2h^2 f''(c) - \frac{4h^3}{3} f^{(3)}(c) + \theta(h^2) \ldots\ldots\ldots\ldots\ldots(5)$ then (5)-2*(4) and collecting terms yields:

$$h^2 f^{(2)}(c) = f(c-2h) - 2f(c-h) + f(c) + h^3 f^{(3)}(c) + \theta(h^2) \ldots\ldots\ldots\ldots\ldots(6)$$

Replacing (2) in (6) and collecting terms yields:

$\boldsymbol{h^2 f^{(2)}(c) = -f(c-3h) + 4f(c-2h) - 5f(c-h) + 2f(c) + \theta(h^2)} \ldots\ldots\ldots\ldots\ldots$ **or**

$$\boldsymbol{h^2 f_j^{(2)} = -f_{j-3} + 4f_{j-2} - 5f_{j-1} + 2f_j + \theta(h^2)} \ldots\ldots\ldots\ldots\ldots$$

Ex2

Analytical solution is: $f'(x) = 10\pi Cos(10\pi.x)$, so $f'(0) = 10\pi Cos(10\pi.0) = 10\pi \cong 31.41$

The forward representation with $\theta(h)$ is: $f_j' = \frac{f_{j+1} - f_j}{h} + \theta(h)$ *or* $f'(c) = \frac{f(c+h) - f(c)}{h} + \theta(h)$

With h=0.2 and c=0 , then

$$f'(0) = \frac{f(0+0.2) - f(0)}{0.2} + \theta(0.2) = \frac{\sin 10\pi(0.2) - \sin 10\pi(0)}{0.2} + \theta(0.2) = 0 + \theta(0.2)$$

The forward representation with $\theta(h^2)$ is: $f_j' = \frac{-f_{j+2} + 4f_{j+1} - 3f_j}{2h} + \theta(h^2)$ or

$$f^{'}(c)=\frac{-f(c+2h)+4f(c+h)-3f(c)}{2h}+\theta(h^2)$$

With h=0.2 and c=0 , then

$$f^{'}(0)=\frac{-f(0+0.4)+4f(0+0.2)-3f(0)}{2(0.2)}+\theta(0.2)^2$$

$$f^{'}(0)=\frac{-\sin 10\pi(0.4)+4\sin 10\pi(0.2)-3\sin 10\pi(0)}{2(0.2)}+\theta(0.2)=0+\theta(0.2)$$

We Remarque that in the two cases the derivative is always zero but the derivative analytically is 31.41 at x=0 so the choice of the length h=0.2 is very bad, so it would be necessary to take h smaller.

Ex3

Solving equations: $x_1=\frac{30-2x_2-3x_3}{8}, x_2=\frac{1-x_1-2x_3}{-9}, x_3=\frac{31-2x_1-3x_2}{6}$ Using initial condition $[x_1,x_2,x_3]=[1,1,1]$

Iterations	x_1	x_2	x_3
0	3.1250	0.4583	3.8959
1	2.1745	0.9963	3.9437
2	2.0220	0.9899	3.9977

Ex4

See lecture

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Numerical Methods
First Exam

Ex1

Solve this linear system by Gauss Elimination.

$$\begin{cases} 3x + 5y + 7z = 101 \\ 2x + 10y + 6z = 134 \\ 1x + 2y + 3z = 40 \end{cases}$$

Ex2 :

Solve this linear system using Gauss Seidel process.

$$\begin{cases} 10x_1 - x_2 + 2x_3 = 6 \\ -x_1 + 11x_2 - x_3 + 3x_4 = 25 \\ 2x_1 - x_2 + 10x_3 - x_4 = -11 \\ 3x_2 - x_3 + 8x_4 = 15 \end{cases}$$

Ex3

Explain the Gauss elimination process……………………………………

Step1: ……………….…..… …………..…..…………. ………………….
Step2: ……………….…..… …………..…..…………. ………………….
Step3: ……………….…..… …………..…..…………. ………………….
Step4: ……………….…..… …………..…..…………. ………………….
Step5: ……………….…..… …………..…..…………. ………………….
Step6: ……………….…..… …………..…..…………. ………………….
Step7: ……………….…..… …………..…..…………. ………………….
Step8: ……………….…..… …………..…..…………. ………………….
Step9: ……………….…..… …………..…..…………. ………………….

Ex4:

Write the program of Simpson’s method

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Solution : ☹

Ex1:(5p)

$$\begin{array}{l} (L1) \\ (L2) \\ (L3) \end{array} \left[\begin{array}{cccccccc} 3x & + & 5y & + & 7z & = & 101 \\ 2x & + & 10y & + & 6z & = & 134 \\ 1x & + & 2y & + & 3z & = & 40 \end{array} \right.$$

The augmented matrix : $\left[\begin{array}{cccc} \boxed{3} & 5 & 7 & 101 \\ 2 & 10 & 6 & 134 \\ 1 & 2 & 3 & 40 \end{array} \right]$

Iteration **1 : (2p)**

$$\begin{array}{l} L1 =- L1 \\ L2 =- 3\,L2 - 2\,L1 \\ L3 =- 3\,L3 - 1\,L1 \end{array} \left[\begin{array}{ccccccc} 3x & + & 5y & + & 7z & = & 101 \\ & & 20y & + & 4z & = & 200 \\ & & 1y & + & 2z & = & 19 \end{array} \right. \begin{array}{l} L1 =- L1 \\ L2 =- 3\,L2 - 2\,L1 \\ L3 =- 3\,L3 - 1\,L1 \end{array} \left[\begin{array}{cccc} 3 & 5 & 7 & 101 \\ 0 & \boxed{20} & 4 & 200 \\ 0 & 1 & 2 & 19 \end{array} \right]$$

Iteration **2 : (2p)**

$$\begin{array}{l} L1 =- L1 \\ L2 =- L2 \\ L3 =- 20\,L3 - 1\,L2 \end{array} \left[\begin{array}{ccccccc} 3x & + & 5y & + & 7z & = & 101 \\ & & 20y & + & 4z & = & 200 \\ & & & & 36z & = & 180 \end{array} \right. \begin{array}{l} L1 =- L1 \\ L2 =- L2 \\ L3 =- 20\,L3 - 1\,L2 \end{array} \left[\begin{array}{cccc} 3 & 5 & 7 & 101 \\ 0 & 20 & 4 & 200 \\ 0 & 0 & 36 & 180 \end{array} \right]$$

(x,y,z)= (7,9,5) **(1p)**

Ex2: (5p)

$10x_1 - x_2 + 2x_3 = 6$ $-x_1 + 11x_2 - x_3 + 3x_4 = 25$ $2x_1 - x_2 + 10x_3 - x_4 = -11$ $3x_2 - x_3 + 8x_4 = 15$	Solving for x_1, x_2, x_3 and x_4 gives:	$x_1 = \frac{x_2}{10} - \frac{x_3}{5} + \frac{3}{5}$ $x_2 = \frac{x_1}{11} + \frac{x_3}{11} - \frac{3x_4}{11} + \frac{25}{11}$ $x_3 = -\frac{x_1}{5} + \frac{x_2}{10} + \frac{x_4}{10} - \frac{11}{10}$ $x_4 = -\frac{3x_2}{8} + \frac{x_3}{8} + \frac{15}{8}$ **(1p)**

Suppose we choose (0, 0, 0, 0) as the initial approximation, then the first approximate solution is given by

$$x_1 = \frac{3}{5} = 0.6$$

$$x_2 = \frac{(\frac{3}{5})}{11} + \frac{25}{11} = \frac{3}{55} + \frac{25}{11} = 2.3272$$

$$x_3 = -\frac{(\frac{3}{5})}{5} + \frac{2.3272}{10} - \frac{11}{10} = -0.9873$$

$$x_4 = -\frac{3(2.3272)}{8} + \frac{(-0.9873)}{8} + \frac{15}{8} = 0.8789$$

Using the approximations obtained, the iterative procedure is repeated until the desired accuracy has been reached. The following are the approximante solutions after four iterations.

x_1	x_2	x_3	x_4	
0.6	2.32727	-0.987273	0.878864	**(0.5)**
1.03018	2.03694	-1.01446	0.984341	**(0.5)**
1.00659	2.00356	-1.00253	0.998351	**(0.5)**
1.00086	2.0003	-1.00031	0.99985	**(0.5)**

The iterative procedure is stopped as soon as the differences between the $x^{(k+1)}$ and $x^{(k)}$ values are suitably small or to end the iteration when

$S = \sum_{i=1}^{n} \left| x_i^{(k+1)} - x_i^{(k)} \right| < \varepsilon$ small number (usually chosen at first) so $S \cong 0.0127$

Finally the approximate solution is (1.00086, 2.0003, -1.00031, 0.99985) **(2p)**

If your solution diverges you must give this remark: The gauss Seidel may diverge from the exact solution so, in order to improve the chance of convergence; the system of equations should be rearranged before applying the process.

Ex 3) (4.5p)

From the flow chart of Gauss elimination

Step1: read data number of equations N **(0.5)**
Step2: read elements of augmented matrix **(0.5)**
Step3: for k=1 to N-1 **(0.5)**
Step4: Search of the largest pivot and memorize the number of this row **(0.5)**
Step5: Is row interchange required? **(0.5)**
Step6: Elimination process **(0.5)**
Step7: Close the loop k **(0.5)**
Step8: Back substitution process **(0.5)**
Step9: Write values of the unknown **(0.5)**

Ex4 (4.5p) (Programming of)

$$\boxed{S(n) = \frac{b-a}{3n}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 4f(x_{n-1}) + f(x_n) \right]}$$

$f(x_0) = f(a)$ and $f(x_n) = f(b)$ with $X_i = a + i.h$, $h = \frac{b-a}{n}$

```
%Simpson integration (for example using Matlab)
a=input('the initial value of the interval a') (0.5)
b=input('the final value of the interval b')  (0.5)
n=input('the lengh') (0.5)
f=inline('sqrt(1+x^2)','x'); (0.5)
h=(b-a)/n  (0.5)
% first loop (0.5)
s1=0
for i=a+h:2*h:b-h
s1=s1+f(i)
end
% second loop (0.5)
s2=0
for j=a+2*h:2*h:b-2*h
   s2=s2+f(j)
end
simp=(h/3)*(f(a)+f(b)+4*s1+2*s2) (1)
```

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DGEE/Promotion EO5 **Wednesday,30/01/2013**

Numerical Methods Exam

Ex1

A) Do three iterations of the bisection method for finding a zero of the function $f(x) = x^3 - 6$. Start with the bracketing interval [1.5, 2.0].
Answer:
Iteration 0: $[\boxed{}, \boxed{}]$ Iteration 1: $[\boxed{}, \boxed{}]$ Iteration 2: $[\boxed{}, \boxed{}]$

B) Do two iterations of the Newton-Raphson method for finding a zero of the function $f(x) = x^3 - 6$. Use the initial value $x_0 = 2$.

Answer: $x_1 = \boxed{}$, $x_2 = \boxed{}$

Ex2 :

Use the formula of Newton-Raphson method (first order) to solve:
$f(x) = x^3 - 0.03x^2 + 2.4\times10^{-6}$

Ex3 :

Let us use Gauss Elimination Method to solve the following system:

$$\begin{cases} x_1 + (1/2)\, x_2 + (1/3)\, x_3 = 11/6 \\ (1/2)\, x_1 + (1/3)\, x_2 + (1/4)\, x_3 = 13/12 \\ (1/3)\, x_1 + (1/4)\, x_2 + (1/5)\, x_3 = 47/60 \end{cases}$$

Ex4:

Consider the data set :

x	-1	0	1	2
y	5	1	1	11

Using Lagrange, find the polynomial to interpolate all the data.

Ex5:

Write a PASCAL program to calculate the area and circumstance of circle by inputting the radius of the circle.

Given:
1. Variables names: radius, pi, area, circumstance with data type : real
2. pi := 3.1416
3. radius:=10 cm.

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Solution:

Ex1:(4 points)

A) Answer:

Iteration 0: [1.5 , 2.0] Iteration 1: [1.75 , 2.0] Iteration 2: [1.75 , 1.875]

B) f(x) $= x^3 - 6$. and $f'(x) = 3x^2$.

Take the initial value $x_0 = 2$. $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

Answer: x_1= 1.833 x_2= 1.817

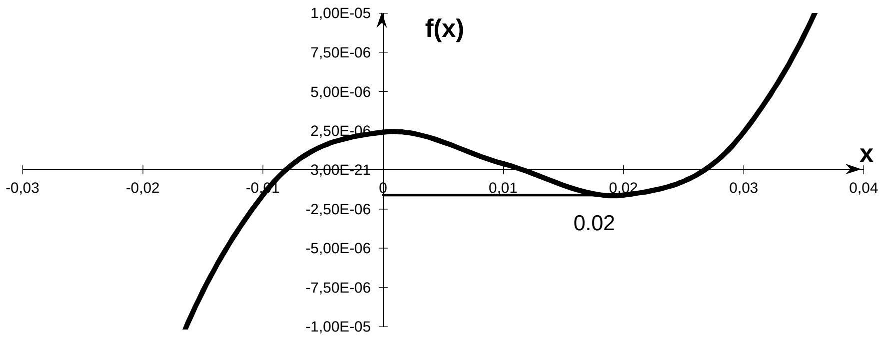
Ex2: (4 points)

The formula of Newton-Raphson method is : $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

$$f(x) = x^3 - 0.03x^2 + 2.4\times10^{-6}$$
$$f'(x) = 3x^2 - 0.06x$$

The solutions are between [0.01, 0.015] and [-0.01, -0.005]

$x_0 = 0$ or $x_0 = 0.02$, division by zero occurs.



Ex 3): (4 points)

$$\begin{cases} x_1 + (1/2)\,x_2 + (1/3)\,x_3 = 11/6 \\ (1/2)\,x_1 + (1/3)\,x_2 + (1/4)\,x_3 = 13/12 \\ (1/3)\,x_1 + (1/4)\,x_2 + (1/5)\,x_3 = 47/60 \end{cases}$$

$m_{21} = 1/2$ and $m_{31} = 1/3$

$$\begin{cases} x_1 + (1/2)\,x_2 + (1/3)\,x_3 = 11/6 \\ 0 + (1/12)\,x_2 + (1/12)\,x_3 = 1/6 \\ 0 + (1/12)\,x_2 + (4/45)\,x_3 = 31/180 \end{cases}$$

$m_{32} = 1$

$$\begin{cases} x_1 + (1/2)\, x_2 + (1/3)\, x_3 = 11/6 \\ 0 + (1/12)\, x_2 + (1/12)\, x_3 = 1/6 \\ 0 + 0 + (1/180)\, x_3 = 1/180 \end{cases}$$

From which we compute $x_3 = 1$ and then, by back substitution, the remaining unknowns $x_2 = 1$ and $x_1 = 1$

Ex4: (4 points)

The polynomial that interpolates all the data is:

$$f(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} f(x_1) + \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} f(x_2)$$
$$+ \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} f(x_3) + \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} f(x_4)$$

$f(x) = x^3 + 2x^2 - 3x + 1$ and $g(x) = \frac{1}{8}x^4 + \frac{3}{4}x^3 + \frac{15}{8}x^2 - \frac{11}{4}x + 1$ both interpolate all the data.

Ex5: (4 points) for programming

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Detail of ex1

(A) $[a,b] = [1.5, 2.0]$ is a bracketing interval of f because the function has different signs at the interval endpoints: $f(a) = f(1.5) = -2.625$ and $f(b) = f(2.0) = 2$. First iteration: the midpoint is $m_1 = \frac{1.5+2.0}{2} = 1.75$, and $f(m_1) = f(1.75) \approx -0.640 < 0$, so m_1 replaces the bracketing interval's left endpoint, and the next bracketing interval is $[\boxed{1.75}, \boxed{2.0}]$. Second iteration: $m_2 = \frac{1.75+2.0}{2} = 1.875$, and $f(m_2) = f(1.875) \approx 0.591 > 0$, so the next bracketing interval is $[\boxed{1.75}, \boxed{1.875}]$.

(B) The Newton-Raphson method is $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$. With $f(x) = x^3 - 6$ we have $f'(x) = 3x^2$ and so $x_{k+1} = x_k - \frac{x_k^3-6}{3x_k^2}$. With the starting value $x_0 = 2$ we obtain

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = x_0 - \frac{x_0^3 - 6}{3x_0^2} = 2 - \frac{2^3 - 6}{3 \cdot 2^2} \approx \boxed{1.833}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = x_1 - \frac{x_1^3 - 6}{3x_1^2} \approx (1.833) - \frac{(1.833)^3 - 6}{3 \cdot (1.833)^2} \approx \boxed{1.817}$$

Detail of ex2

The formula of Newton-Raphson method is: $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

Consequently if an iteration value, x_i is such that $f'(x_i) \cong 0$, then one can face division by zero or a near-zero number. This will give a large magnitude for the next value, x_{i+1}. An example is finding the root of the equation

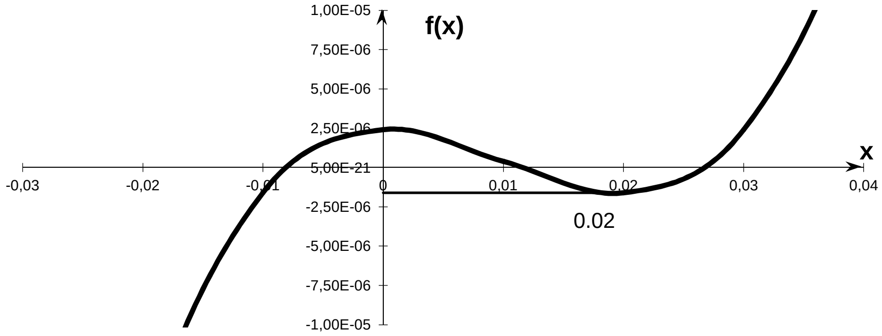
$$f(x) = x^3 - 0.03x^2 + 2.4 \times 10^{-6}$$

in which case

$$f'(x) = 3x^2 - 0.06x$$

For $x_0 = 0$ or $x_0 = 0.02$, division by zero occurs (Figure 3.14). For an initial guess close to 0.02, of $x_0 = 0.01999$, even after 9 iterations, the Newton-Raphson method is not converging.

Iteration Number	x_i	$\lvert\varepsilon_a\rvert$	$f(x_i)$
0	0.019990	—	-1.6000×10^{-6}
1	-2.6480	100.75	-18.778
2	-1.7620	50.282	-5.5638
3	-1.1714	50.422	-1.6485
4	-0.77765	50.632	-0.48842
5	-0.51518	50.946	-0.14470
6	-0.34025	51.413	-0.042862
7	-0.22369	52.107	-0.012692
8	-0.14608	53.127	-0.0037553
9	-0.094490	54.602	-0.0011091



f(x)
x
1,00E-05
7,50E-06
5,00E-06
2,50E-06
5,00E-21
-2,50E-06
-5,00E-06
-7,50E-06
-1,00E-05
-0,03
-0,02
-0,01
0
0,01
0,02
0,03
0,04
0.02

Numerical Methods
Second Mid-term Exam

Ex1

Consider the specific heat of water as a function of temperature at 1.0 atm.
We have the following data:

Temp (C)	20	25	30	35	40	45	50
Heat(Cal /g/C)	0.99883	0.99828	0.99802	0.99795	0.99804	0.99826	0.99854

we wish to estimate the heat at x = 37. It is useful to practice some judgment here: just because there is a table available with lots of data does not necessarily imply that they should all be used to get an idea of what is happening around a specific point. What we could do, for example.

Ex2:

Prove the statements:

$$\delta^3 f_j = f_{j+\frac{3}{2}} - 3f_{j+\frac{1}{2}} + 3f_{j-\frac{1}{2}} - f_{j-\frac{3}{2}}$$

Ex3:

Given that f (-2) = 46, f (-1) = 4, f (1) = 4, f (3) = 156, and f (4) = 484, use **Lagrange's interpolation formula** to estimate the value of f(0).

Ex4:

Estimate by **numerical integration**.the value of the integral $\int_0^1 \frac{1}{1+x}dx$ with h=0.1 and bonded the error.

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Solution :

Ex1:

We construct a cubic (n = 3) polynomial using four specific points near x = 37.

We choose $x_0=30, x_1=35, x_2=40, x_4=45$ Then we have:

$$L_0(x) = \frac{(x-35)(x-40)(x-45)}{(30-35)(30-40)(30-45)}$$
$$L_1(x) = \frac{(x-30)(x-40)(x-45)}{(35-30)(35-40)(35-45)}$$
$$L_2(x) = \frac{(x-30)(x-35)(x-45)}{(40-30)(40-35)(40-45)}$$
$$L_3(x) = \frac{(x-30)(x-35)(x-40)}{(45-30)(45-35)(45-40)}$$

Thus, the cubic polynomial we want is:
$p(x) = 0.99802L_0(x) + 0.99795L_1(x) + 0.99804L_2(x) + 0.99826L_3(x)$
Evaluating the cubic interpolant at x = 37, we end

$$L_0(37) = \frac{(2)(-3)(-8)}{(-5)(-10)(-15)} = 0.064000$$
$$L_1(37) = \frac{(7)(-3)(-8)}{(5)(-5)(-10)} = 0.672000$$
$$L_2(37) = \frac{(7)(2)(-8)}{(10)(5)(-5)} = 0.448000$$
$$L_3(37) = \frac{(7)(2)(-3)}{(15)(10)(5)} = -0.056000$$

Summing , we get p(37) = 0.99797. This is comparable to the value of 0.99799 obtained by linear interpolation of the two closest neighbors, at 35 and 40.

Ex2:

$$\begin{aligned}
\delta^3 f_j &= \delta^2(\delta f_j) \\
&= \delta^2(f_{j+\frac{1}{2}} - f_{j-\frac{1}{2}}) \\
&= \delta(\delta f_{j+\frac{1}{2}} - \delta f_{j-\frac{1}{2}}) \\
&= \delta(f_{j+1} - 2f_j + f_{j-1}) \\
&= f_{j+\frac{3}{2}} - 3f_{j+\frac{1}{2}} - 3f_{j-\frac{1}{2}} - f_{j-\frac{3}{2}}.
\end{aligned}$$

Ex3:

The Lagrange coefficients are

$$L_0(x) = \frac{(x+1)(x-1)(x-3)(x-4)}{(-1)(-3)(-5)(-6)} \qquad \text{for } x_0 = -2,$$

$$L_1(x) = \frac{(x+2)(x-1)(x-3)(x-4)}{1(-2)(-4)(-5)} \qquad \text{for } x_1 = -1,$$

$$L_2(x) = \frac{(x+2)(x+1)(x-3)(x-4)}{3\times 2(-2)(-3)} \qquad \text{for } x_2 = 1,$$

$$L_3(x) = \frac{(x+2)(x+1)(x-1)(x-4)}{5\times 4\times 2(-1)} \qquad \text{for } x_3 = 3,$$

$$L_4(x) = \frac{(x+2)(x+1)(x-1)(x-3)}{6\times 5\times 3\times 1} \qquad \text{for } x_4 = 4.$$

Thus

$$\begin{aligned} f(0) &= L_0(0)\times 46 + L_1(0)\times 4 + L_2(0)\times 4 + L_3(0)\times 156 \\ &\qquad + L_4(0)\times 484 \\ &= (-92 + 36 + 40 - 468 + 484)/15 \\ &= 0 \end{aligned}$$

Ex4:

We have

$$f(x) = \frac{1}{1+x}, \quad f''(x) = \frac{2}{(1+x)^3}, \quad f^{(4)}(x) = \frac{24}{(1+x)^5}.$$

x	0	0·1	0·2	0·3	0·4	0·5
$f(x)$	1·000000	0·909091	0·833333	0·769231	0·714286	0·666667
x	0·6	0·7	0·8	0·9	1·0	
$f(x)$	0·625000	0·588235	0·555556	0·526316	0·500000	

By Simpson's rule,

$$\begin{aligned} \int_0^1 \frac{1}{1+x}\,dx = \frac{0·1}{3}[1 &+ 4(0·909091 + 0·769231 + 0·666667 \\ &+ 0·588235 + 0·526316) \\ &+ 2(0·833333 + 0·714286 + 0·625000 \\ &+ 0·555556) + 0·500000] \\ = 0·6931(5). \end{aligned}$$

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Numerical Methods
Second Mid-term Exam

Ex1

Use Taylor series to find the truncation errors in the formulae:

- $f'(x_j) \approx \left[f(x_j + h) - f(x_j) \right] / h$
- $f'(x_j + \frac{1}{2}h) \approx \left[f(x_j + h) - f(x_j) \right] / h$
- $f''(x_j) \approx \left[f(x_j + 2h) - 2f(x_j + h) + f(x_j) \right] / h^2$

Ex2

Obtain an estimate of the integral $\int_{0.1}^{0.3} e^x dx$ using the trapezoidal rule with h=0.2 , 0.1 , 0.05

Ex3

Given that f (0) = 2.3913, f(1) = 2.3919, f (3) = 2.3938, and f (4) = 2.3951, use **Lagrange's** interpolation formula to estimate the value of f(2).

Ex4

Give an algorithm of D=A[i,j] . B[j,k] + C[i,k] with i=1:n , j=1:m and k=1:l, give the program using Pascal or Fortran.

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Solution :

Ex1:

i) **Expanding about $x = x_j$:**

$f(x_j + h) = f(x_j) + hf'(x_j) + \frac{1}{2}h^2 f''(x_j) + \dots,$

so $(f(x_j + h) - f(x_j))/h = f'(x_j) + \frac{1}{2}hf''(x_j) + \dots$

and the error $\approx \frac{1}{2}hf''(x_j)$.

ii) **Expanding about $x = x_j + \frac{1}{2}h$:**

$$f(x_j + h) = f(x_j + \tfrac{1}{2}h) + \tfrac{1}{2}hf'(x_j + \tfrac{1}{2}) + \tfrac{1}{8}h^2 f''(x_j + \tfrac{1}{2}h) + \tfrac{1}{48}h^3 f'''(x_j + \tfrac{1}{2}h) + \dots$$

and $f(x_j) = f(x_j + \frac{1}{2}h) - \frac{1}{2}hf'(x_j + \frac{1}{2}h) + \frac{1}{8}h^2 f''(x_j + \frac{1}{2}h) - \frac{1}{48}h^3 f'''(x_j + \frac{1}{2}h) + \dots$

so $(f(x_j + h) - f(x_j))/h = f'(x_j + \frac{1}{2}h) + \frac{1}{24}h^2 f'''(x_j + \frac{1}{2}h) + \dots$

and the error $\approx \frac{1}{24}h^2 f'''(x_j + \frac{1}{2}h)$.

iii) **Expanding about $x = x_j$:**

$f(x_j + 2h) = f(x_j) + 2hf'(x_j) + 2h^2 f'''(x_j) + \frac{4}{3}h^3 f'''(x_j) + \dots,$

so $(f(x_j + 2h) - 2f(x_j + h) + f(x_j))/h^2 = f''(x_j) + hf'''(x_j) + \dots$

and the error $\approx hf'''(x_j)$.

Ex2

If we use *T(h)* to denote the approximation with strip width *h*, we obtain

$$T(0.2) = \frac{0.2}{2}[1.10517 + 1.34986] = 0.24550$$

$$T(0.1) = \frac{0.1}{2}[1.10517 + 2(1.22140) + 1.34986] = 0.24489$$

$$T(0.05) = \frac{0.05}{2}[1.10517 + 2(1.16183 + 1.22140 + 1.28403) + 1.34986] = 0.24474$$

Ex3:

Lagrange's interpolation formula is:

$$L_0(x)=\frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)(0-4)}=-(1/12)(x-1)(x-3)(x-4)$$

$$L_1(x)=\frac{(x-0)(x-3)(x-4)}{(1-0)(1-3)(1-4)}=(1/6)(x-0)(x-3)(x-4)$$

$$L_2(x)=\frac{(x-0)(x-1)(x-4)}{(3-0)(3-1)(3-4)}=-(1/6)(x-0)(x-1)(x-4)$$

$$L_3(x)=\frac{(x-0)(x-1)(x-3)}{(4-0)(4-1)(4-3)}=(1/12)(x-0)(x-1)(x-3)$$

$$F\ (x)=-(1/12)(x-1)(x-3)(x-4)*2.3913+(1/6)(x-0)(x-3)(x-4)*2.3919$$
$$-(1/6)(x-0)(x-1)(x-4)*2.3938+(1/12)(x-0)(x-1)(x-3)*2.3951$$

$$F\ (x)=-(1/12)(2-1)(2-3)(2-4)*2.3913+(1/6)(2-0)(2-3)(2-4)*2.3919$$
$$-(1/6)(2-0)(2-1)(2-4)*2.3938+(1/12)(2-0)(2-1)(2-3)*2.3951$$

$$F\ (x)=-(2/12)*2.3913+(4/6)*2.3919+(4/6)*2.3938-(2/12)*2.3951=2.3927$$

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Numerical Methods Second Exam

Ex1: (4p)

Estimate the value of the integral $\int_{1.0}^{1,3} \sqrt{x}\,dx$

Ex2: (6p)

Give the algorithm and the program of Lagrange interpolation

Ex3: (4p)

Prove the statements:

$$\delta^3 f_j = f_{j+\frac{3}{2}} - 3f_{j+\frac{1}{2}} + 3f_{j-\frac{1}{2}} - f_{j-\frac{3}{2}}$$

Ex4: (6p)

For each of :

- Euler's method (first order):
- Taylor series (fourth order):

state the **main disadvantage**?

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Solution :

Ex1:

With $b-a = 1{\cdot}30 - 1{\cdot}00 = 0{\cdot}30$, we may choose

$$h = 0{\cdot}30,\ 0{\cdot}15,\ 0{\cdot}10,\ 0{\cdot}05,\ \ldots .$$

If $T(h)$ denotes the approximation corresponding to strip width h, we get

$$T(0{\cdot}30) = \frac{0{\cdot}30}{2}(1{\cdot}00000 + 1{\cdot}14018) = 0{\cdot}32102(7),$$

$$T(0{\cdot}15) = \frac{0{\cdot}15}{2}(1{\cdot}00000 + 1{\cdot}14018) + (0{\cdot}15)(1{\cdot}07238)$$

$$= 0{\cdot}16051(4) + 0{\cdot}16085(7) = 0{\cdot}32137(1),$$

$$T(0{\cdot}10) = \frac{0{\cdot}10}{2}(1{\cdot}00000 + 1{\cdot}14018) + (0{\cdot}10)(1{\cdot}04881 + 1{\cdot}09545)$$

$$= 0{\cdot}10700(9) + 0{\cdot}21442(6) = 0{\cdot}32143(5),$$

$$T(0{\cdot}05) = \frac{0{\cdot}05}{2}(1{\cdot}0000 + 1{\cdot}14018) +$$

$$+ (0{\cdot}05)(1{\cdot}02470 + 1{\cdot}04881 + 1{\cdot}07238 + 1{\cdot}09545 + 1{\cdot}11803)$$

$$= 0{\cdot}05350(5) + 0{\cdot}26796(9) = 0{\cdot}32147(4)$$

the answer is in fact $0{\cdot}32148537$, so the error sequence $0{\cdot}00045(8)$, $0{\cdot}00011(4)$, $0{\cdot}00005(0)$, $0{\cdot}00001(1)$

Ex2:

Donne on Lab

Ex3:

$$\begin{aligned}\delta^3 f_j &= \delta^2(\delta f_j)\\ &= \delta^2(f_{j+\frac{1}{2}} - f_{j-\frac{1}{2}})\\ &= \delta(\delta f_{j+\frac{1}{2}} - \delta f_{j-\frac{1}{2}})\\ &= \delta(f_{j+1} - 2f_j + f_{j-1})\\ &= f_{j+\frac{3}{2}} - 3f_{j+\frac{1}{2}} - 3f_{j-\frac{1}{2}} - f_{j-\frac{3}{2}}.\end{aligned}$$

Ex4:

- In **Euler's method** the step length h is very small, the **truncation error** will be large and the results inaccurate.
- In **Taylor series** the **truncation error** is small and the results is best approximative but the method is more complexe for computer implementation

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Sunday, 04 January 2007

Numerical Methods Second Exam

Ex1: (4p)

Evaluate the given code fragments for the various starting values given on the right. Use Matlab to check your answers

1. if n > 1 m = n + 2 else m = N - 2 end	a) n = 7 b) n = 0 c) n = -7	m = ? m = ? m = ?
2. if s <= 1 t = 2z elseif s =10 t = 9 – z elseif s < 100 t = Sqrt(s) else t = s end	a) s = 1 b) s = 7 c) s = 57 d) s = 300	t = ? t = ? t = ? t = ?
3. if t >= 24 z = 3*t + 1 elseif t < 9 z = t^2/3 - 2t else z = Sin(t) end	a) t = 50 b) t = 19 c) t = -6 d) t = 0	h = ? h = ? h = ? h = ?
4. if 0 < x < 7 y = 4x elseif 7 < x < 55 y = -10x else y = = 333 end	a) x = -1 b) x = 5 c) x = 30 d) x = 56	y = ? y = ? y = ? y = ?

Ex2: (6p)

Give the program of Lagrange interpolation

Ex3: (4p)

Write the program to finding the $\sqrt{8}$ using Newton first order

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Ex4: (6p)

For each of :

- Euler's method (first order):
- Taylor series (fourth order):

state the **main disadvantage**?

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Solution :

Ex1:

Ex2:

Donne on Lab

Ex3:

Ex4:

- In **Euler's method** the step length *h* is very small, the **truncation error** will be large and the results inaccurate.
- In **Taylor series** the **truncation error** is small and the results is best approximative but the method is more complexe for computer implementation

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Numerical Methods
Synthesis Exam

Ex1

Write the Pascal program:

1) For finding the mathematical root of the expression $Ax^2 + Bx + C$
2) To describe the weather according to the following temperature classifications:
   greater than 75 hot
   50 to 75 cool
   35 to 49 cold
   less than 35 freezing

3) The code that reads the number of the month and tells the number of days in that month using CASE construct :

Ex2 :

1. Find the inverses of the following matrix, using **elimination and back-substitution**:

$$\begin{bmatrix} 1.3 & 4.6 & 3.1 \\ 5.6 & 5.8 & 7.9 \\ 4.2 & 3.2 & 4.5 \end{bmatrix}$$

2. Use the bisection method to find to 3D the positive root of the equation

$$x - 0.2\sin x - 0.5 = 0.$$

Ex3

1. What is the **geometrical interpretation** of the Newton-Raphson iterative procedure?

2. What is the **convergence criterion** for the Newton-Raphson method?

Ex4:

obtain the estimate of $\sqrt[3]{20}$ from the points (0, 0), (1,1), (8, 2), (27, 3), and (64, 4)

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Solution :

Ex1:

1)

```
a=input('value of a')
 b=input('value of b')
 c=input('value of c')
if a==0
    x=-b/2*a
end
if (a~=0)
del=b^2-4*a*c
if del==0
    x=-b/2*a
elseif del>0
    x1=(-b-sqrt(del))/2*a
    x2=(-b+sqrt(del))/2*a
else
    x1=(-b-sqrt(del))/2*a
    x2=(-b+sqrt(del))/2*a
end
end
```

2)

3)

CASE Month OF
1,3,5,7,8,10,12 : Days := 31;
4,6,9,11 : Days := 30;
2 : Days := 28;
END;

Ex2

1)

	1.3 4.6 3.1 5.6 5.8 7.9 4.2 3.2 4.5
M_{21}= 4.30 M_{31}=3.23	1.3 4.6 3.1 0 -13.98 -5.43 0 -11.658 -5.513
M_{32}=0.83	1.3 4.6 3.1 0 -13.98 -5.43 0 0 -1.006

inv =

0.0460	-0.6053	1.0309
0.4481	-0.4026	0.3981
-0.3616	0.8512	-1.0230

2)

1. In Step 6 we saw that the root lies in the interval $(-0{\cdot}75, -0{\cdot}7)$. Successive bisections produce the following sequence of intervals containing the root: $(-0{\cdot}75, -0{\cdot}725)$, $(-0{\cdot}75, -0{\cdot}7375)$, $(-0{\cdot}74375, -0{\cdot}7375)$.
   Thus the root is $-0{\cdot}74$ to $2D$.
2. Root is $0{\cdot}615$ to 3D.

Ex3:

1)

The geometrical interpretation is that each iteration provides the point at which the tangent at the original point cuts the x-axis. Thus the equation of the tangent at $(x_n, f(x_n))$ is : $y - f(x_0) = f'(x_0)(x - x_0)$ so that $(x_1, 0)$ corresp to $-f(x_0) = f'(x_0)(x_1 - x_0)$,

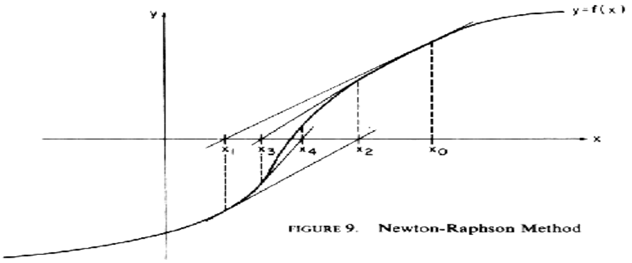

FIGURE 9. Newton-Raphson Method

2) $\Delta x = x_{n+1} - x_n$

Ex4:

$$f(x) \approx \frac{x(x-8)(x-27)(x-64)}{1(-7)(-26)(-63)} \times 1 + \frac{x(x-1)(x-27)(x-64)}{8(7)(-19)(-56)} \times 2 + \frac{x(x-1)(x-8)(x-64)}{27(26)(19)(-37)} \times 3 + \frac{x(x-1)(x-8)(x-27)}{64(63)(56)(37)} \times 4$$

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Numerical Methods
Second Exam

Ex1

1. what is the advantage of Lagrange interpolation ? and give the general form of Lagrange
2. How are the **forward, backward, and central difference obtained**?
3. When are the **forward, backward, and central difference** likely to be of special use?
4. In one line compare between the method of integration that we have seen

Ex2 :

```
Complete the program of Lagrange interpolation
n=input('give the degree of n=')
a=input('give the value of x=')
for i=1:n+1
    x(i)=input('enter your points x(i)=')
end
for i=1:n+1
    f(i)=input('enter the values of f(i)=')
end
l=inline('(x-y)/(z-y)','x','y','z');
L=0;
for k=1:n+1
    p(k)=1;
    for j=1:n+1
        if j~=k
           p(k)..........................................;
        end
    end
    m=p(k)*f(k);
    ..................................;
 end
L
```

Ex3

The values of f(x) are given below for different values of x. find the values of f(23.9)

x	10	15	18	20	22	25	30
F(x)	0.5	0.22	0.32	0.42	0.28	0.25	0.20

Using:

$$f(x) = f(0) + \frac{x}{h}\Delta f_0 + \frac{x(x-h)}{2!.h^2}\Delta^2 f_0 + \frac{x(x-h)(x-2h)}{3!.h^3}\Delta^3 f_0 + ...$$

$$f(x) = f(0) + \frac{x}{h}\nabla f_0 + \frac{x(x+h)}{2!.h^2}\nabla^2 f_0 + \frac{x(x+h)(x+2h)}{3!.h^3}\nabla^3 f_0 + ...$$

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Ex1

1) Lagrange interpolation does not require function values at equal intervals. And is simple to be implemented in computer.

$$P_N(x) = \sum_{k=0}^{N} y_k L_{N,k}(x) \text{ with } L_{N,k}(x) = \frac{\prod_{\substack{j=0 \\ j\neq k}}^{N}(x - x_j)}{\prod_{\substack{j=0 \\ j\neq k}}^{N}(x_k - x_j)}$$

2) backward differences, forward differences and central differences

3) $\Delta f_j = f_{j+1} - f_j$, $\nabla f_j = f_j - f_{j-1}$, $\delta f_j = f_{j+1} - f_{j-1}$

4) Forward differences are useful near the start of a table.

- Central differences are useful away from the ends of a table.
- Backward differences are useful near the end of a table.

Ex2 :

```
p(k)=p(k)*l(a,x(j),x(k));
L=L+m;
```

Ex3

We have :

x	Y=sin(x)	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
10	0.5				
15	0.22	-0.28			
20	0.42	0.2	0.48		
25	0.25	-0.17	-0.37	-0.85	
30	0.20	-0.05	0.12	0.49	1.34

Putting these values in Newton's forward interpolation:

$$f(x) = f(0) + \frac{x}{h}\nabla f_0 + \frac{x(x+h)}{2!.h^2}\nabla^2 f_0 + \frac{x(x+h)(x+2h)}{3!.h^3}\nabla^3 f_0 + ...$$

$$f(x) = -0.17 + -0.37 * x + \frac{x(x+1)}{2!}(-0.85)$$

With x=(x-x_0)/h=(25-23.9)/1=1.1

$$f(23.9) = -0.17 + -0.37 * (1.1) + \frac{1.1(1.1+1)}{2!}(-0.85) = 1.56$$

Numerical Methods
Second Mid-term Exam

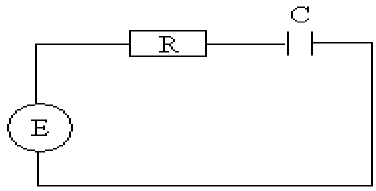
Ex1

Joe had decided to buy stocks of a particularly promising Internet company. The price per share was \$100, and Joe subsequently recorded the stock price at the end of each week. With the abscissae measured in days, the following data were acquired:
(0; 100); (7; 98); (14; 101); (21; 50); (28; 51); (35; 50).
In attempting to analyze what happened, it was desired to approximately evaluate the stock price a few days before the crash.

Pass a linear interpolant through the points with abscissae 7 and 14. Then add to this data set the value at 0 and (separately) the value at 21 to obtain two quadratic interpolants. Evaluate all three interpolants at x = 12. Which one do you think is the most accurate? Explain.

Ex2:

When a capacitor of capacitance C is being charged through a resistor R by a battery which supplies a constant voltage E with amplitude E_0 in series , the instantaneous charge q deposited on the capacitor satisfies the equation relating resistance , capacitor charge and voltage:



Give the algorithm that we need to calculate the capacitor q which is initially uncharged. (q(0)=0)

Ex3:

Calculate this integral using trapezoidal rule : $I = \int_0^\pi \sin(x)dx$ and evaluate the error.

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Solution :

Ex1: (8pts)

We construct : T1: (7 ;98) ;(14 ;101)
T2: (0 ;100) ; (7 ;98) ;(14 ;101)
T3: (7 ;98) ;(14 ;101) ;(21 ;50)

$$f_1(x)=\frac{(x-14)}{(7-14)}.98+\frac{(x-7)}{(14-7)}.101$$

$$f_2(x)=\frac{(x-7)(x-14)}{(0-7)(0-14)}.100+\frac{x(x-14)}{(7)(7-14)}.98+\frac{x(x-7)}{(14)(14-7)}.101$$

$$f_3(x)=\frac{(x-14)(x-21)}{(7-14)(7-21)}.98+\frac{(x-7)(x-21)}{(14-7)(14-21)}.101+\frac{(x-7)(x-14)}{(21-7)(21-14)}.50$$

So: $f_1(x)=0.43x+95.06$

$f_2(x)=0.05x^2-0.63x+99.96$

$f_3(x)=-0.55x^2+11.99x+41$

For x=12 we get : $f_1(12)=100.14$ 2 , $f_2(12)=99.6$ 2 , $f_3(12)=105.65$ 2

The most accurate is T_1 and T_2 2

Ex2: (6pts)

$$\mathrm{R}\frac{\mathrm{dq}}{\mathrm{dt}}+\frac{\mathrm{q}}{\mathrm{C}}=\mathrm{E}\ \Rightarrow R\big(sq(s)-q(0)\big)+\frac{1}{C}q(s)=E(s)$$ the capacitor is initially uncharged then q(0)=0 ,

$$\frac{\mathrm{dq}}{\mathrm{dt}}+\frac{\mathrm{q}}{\mathrm{RC}}=\frac{\mathrm{E}}{\mathrm{R}}$$ so $$\mathrm{q}'=\frac{\mathrm{E}}{\mathrm{R}}-\frac{\mathrm{q}}{\mathrm{RC}}$$ 4 or $\mathrm{q}'=\mathrm{f(t,q)}$

A)Euler method :(if we use this method)

$y_{n+1}=y_n+h.f(x_n,y_n)$ so $q_{n+1}=q_n+h.\left(\frac{\mathrm{E}}{\mathrm{R}}-\frac{\mathrm{q_n}}{\mathrm{RC}}\right)$ 2

B)Runge kutta method : :(if we use this method)

$$\begin{cases}\mathrm{k_1=h.f(t_n,q_n)}\\ \mathrm{k_2=h.f(t_n+h,q_n+k_1)}\\ \mathrm{q_{n+1}=q_n+\frac{1}{2}(k_1+k_2)}\end{cases}$$ so $$\begin{cases}\mathrm{k_1=h.(\frac{E}{R}-\frac{q_n}{RC})}\\ \mathrm{k_2=h.(\frac{E}{R}-\frac{q_n}{RC}-\frac{k_1}{RC})}\\ \mathrm{q_{n+1}=q_n+\frac{1}{2}(k_1+k_2)}\end{cases}$$

Ex3: (6pts)

A) If n=3 :

$$\mathrm{I}=\frac{\pi/3}{2}\left[f(0)+2f(\pi/3)+2f(2\pi/3+f(\pi)\right]=\frac{\pi/3}{2}\left[\sin(0)+2\sin(\pi/3)+2\sin(2\pi/3+\sin(\pi)\right]=$$ 4

1.8137

The real integral is $\mathrm{I}=2$ so the error is E=0.1863 so 9% 2

B) if n=4 : $$\mathrm{I}=\frac{\pi/4}{2}\left[f(0)+2f(\pi/4)+2f(\pi/2)+2f(3\pi/4)+f(\pi)\right]$$

$$=\frac{\pi/4}{2}\left[\sin(0)+2\sin(\pi/4)+2\sin(\pi/2)+2\sin(3\pi/4)+\sin(\pi)\right]=\frac{\sqrt{2}}{3}\pi=1.8961$$

The real integral is $\mathrm{I}=2$ so the error is E=0.1039 so 5%

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Numerical Methods
Make up Exam

Ex1:

Numerically approximate the integral $\int_0^3 (3e^{-x} si(x^2)+1)dx$ by using the trapezoidal rule with n = 1, 2, 4, 8, and 16 subintervals.

Ex2:

Check the inverse of this matrix A:

$$A=\begin{bmatrix} 0.20 & 0.24 & 0.12 \\ 0.10 & 0.24 & 0.24 \\ 0.05 & 0.30 & 0.49 \end{bmatrix}$$

Ex3:

Prove the statements:

$$\delta^3 f_j = f_{j+\frac{3}{2}} - 3f_{j+\frac{1}{2}} + 3f_{j-\frac{1}{2}} - f_{j-\frac{3}{2}}$$

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Numerical Methods
Make up Exam

Ex1:

Use the **bisection method** to find the root of the equation : x+cosx = 0.

Ex2:

Solve the following system by Gauss elimination:

$$x_1 + x_2 - x_3 = 0,$$

$$2x_1 - x_2 + x_3 = 6,$$

$$3x_1 + 2x_2 - 4x_3 = -4.$$

Ex3:

Find the inverse of the following matrix, using **elimination and back-substitution**

$$\begin{bmatrix} 2 & 6 & 4 \\ 6 & 19 & 12 \\ 2 & 8 & 14 \end{bmatrix}$$

Ex4:

Estimate the value of the integral $\int_{1.0}^{1,3} \sqrt{x}dx$

Ex5:

Give the algorithm and the program of Lagrange interpolation

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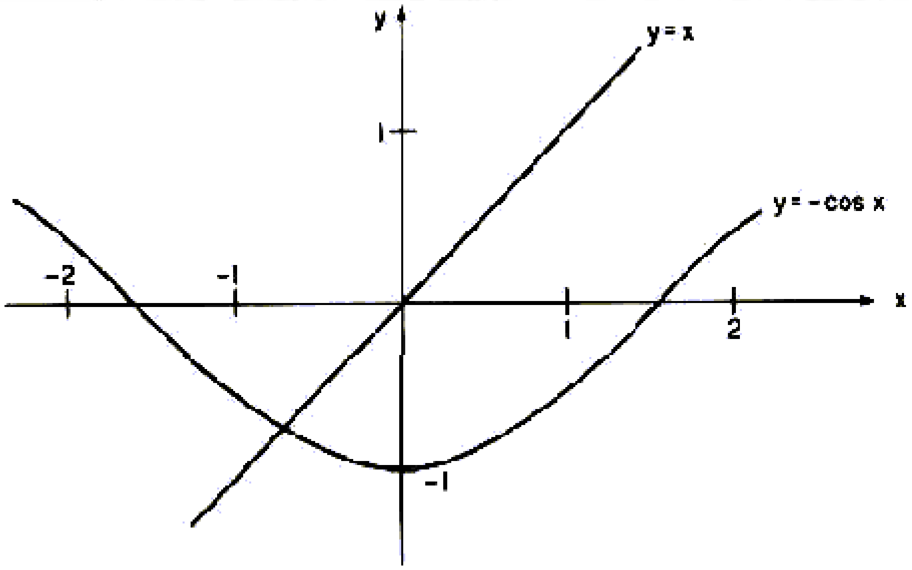
Numerical Methods
Make up (Solution)

Ex1:

The curves are sketched in Fig below one real root near $x = -0{\cdot}7$.
Tabulating confirms this:

x	$-0{\cdot}7$	$-0{\cdot}8$	$-0{\cdot}75$
$\cos x$	$0{\cdot}7648$	$0{\cdot}6967$	$0{\cdot}7317$
$x + \cos x$	$0{\cdot}0648$	$-0{\cdot}1033$	$-0{\cdot}0183$



Graphs of $y = x$ and $y = -\cos x$

Ex2:

m	Augmented Matrix				Check
	1	1	−1	0	1
	2	−1	1	6	8
	3	2	−4	−4	−3
	1	1	−1	0	1
−2		−3	3	6	6
−3		−1	−1	−4	−6
	1	1	−1	0	1
		−3	3	6	6
−1/3			−2	−6	−8

Solution by back-substitution		*Residuals*
$-2x_3 = -6$	$\to x_3 = 3$	$0-(2+1-3)=0$
$-3x_2+9 = 6$	$\to x_2 = 1$	$6-(4-1+3)=0$
$x_1+1-3 = 0$	$\to x_1 = 2$	$-4-(6+2-12)=0$

Ex3:

m	**A**			**I**			*Check*	*Row operation*
	2	6	4	1	0	0	13	(1)
	6	19	12	0	1	0	38	(2)
	2	8	14	0	0	1	25	(3)
	2	6	4	1	0	0	13	(4) = (1)
−3	0	1	0	−3	1	0	−1	(5) = (2) − 3(1)
−1	0	2	10	−1	0	1	12	(6) = (3) − 1(1)
	2	6	4	1	0	0	13	(7) = (1)
	0	1	0	−3	1	0	−1	(8) = (5)
−2	0	0	10	5	−2	1	14	(9) = (6) − 2(5)
Inverse matrix				8·5	−2·6	−0·2		
				−3	1	0		(Check that $AA^{-1} = I$)
				0·5	−0·2	0·1		

Note: The first column of A^{-1} is $\begin{pmatrix} u_3 \\ u_2 \\ u_1 \end{pmatrix}$, found by solving

$$\begin{bmatrix} 2 & 6 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} u_3 \\ u_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 5 \end{bmatrix} \text{ by back-substitution.}$$

The second column is found by solving

$$\begin{bmatrix} 2 & 6 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} u_6 \\ u_5 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix};$$

the third is from

$$\begin{bmatrix} 2 & 6 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} u_9 \\ u_8 \\ u_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Ex4:

With $b-a = 1{\cdot}30 - 1{\cdot}00 = 0{\cdot}30$, we may choose

$$h = 0{\cdot}30, 0{\cdot}15, 0{\cdot}10, 0{\cdot}05, \dots .$$

If $T(h)$ denotes the approximation corresponding to strip width h, we get

$$T(0{\cdot}30) = \frac{0{\cdot}30}{2}(1{\cdot}00000 + 1{\cdot}14018) = 0{\cdot}32102(7),$$

$$T(0{\cdot}15) = \frac{0{\cdot}15}{2}(1{\cdot}00000 + 1{\cdot}14018) + (0{\cdot}15)(1{\cdot}07238)$$

$$= 0{\cdot}16051(4) + 0{\cdot}16085(7) = 0{\cdot}32137(1),$$

$$T(0{\cdot}10) = \frac{0{\cdot}10}{2}(1{\cdot}00000 + 1{\cdot}14018) + (0{\cdot}10)(1{\cdot}04881 + 1{\cdot}09545)$$

$$= 0{\cdot}10700(9) + 0{\cdot}21442(6) = 0{\cdot}32143(5),$$

$$T(0{\cdot}05) = \frac{0{\cdot}05}{2}(1{\cdot}0000 + 1{\cdot}14018) +$$

$$+ (0{\cdot}05)(1{\cdot}02470 + 1{\cdot}04881 + 1{\cdot}07238 + 1{\cdot}09545 + 1{\cdot}11803)$$

$$= 0{\cdot}05350(5) + 0{\cdot}26796(9) = 0{\cdot}32147(4)$$

the answer is in fact $0{\cdot}32148537$, so the error sequence $0{\cdot}00045(8)$, $0{\cdot}00011(4)$, $0{\cdot}00005(0)$, $0{\cdot}00001(1)$

Ex5:

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DGEE/Promotion EO5 **Tuesday,Sept 09,2008**

Numerical Methods
MakeUp Exam

Ex1

Solve the following system by Gauss elimination:

$5.6\,x + 3.8\,y + 1.2\,z = 1.4$

$3.l\,x + 7.l\,y - 4.7\,z = 5.1$

$1.4\,x - 3.4y + 8.3\,z = 2.4$

Ex2

Give the algorithm and program of Gauss elimination

Ex3

Give the algorithm and program of Gauss Seidel

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Solution :

Ex1:

m	*Augmented Matrix*				*Check*
	5·6	3·8	1·2	1·4	12·0
	3·1	7·1	−4·7	5·1	10·6
	1·4	−3·4	8·3	2·4	8·7
	5·6	3·8	1·2	1·4	12·0
−0·554		4·99	−5·36	4·32	3·95
−0·250		−4·35	8·00	2·05	5·70
	5·6	3·8	1·2	1·4	12·0
		4·99	−5·36	4·32	3·95
+0·872			3·33	5·82	9·14 (9·15)

Working to 2*D* (rounded)

Solution by back-substitution

$$\left.\begin{aligned} 3{\cdot}33z &= 5{\cdot}83 \rightarrow z = 1{\cdot}75 \\ 4{\cdot}99y - 5{\cdot}36 \times 1{\cdot}75 &= 4{\cdot}32 \rightarrow y = 2{\cdot}75 \\ 5{\cdot}6x + 3{\cdot}8 \times 2{\cdot}75 + 1{\cdot}2 \times 1{\cdot}75 &= 1{\cdot}4 \rightarrow x = -1{\cdot}99 \end{aligned}\right\}$$

Residuals

$$\begin{aligned} 1{\cdot}4 - (-11{\cdot}14 + 10{\cdot}45 + 2{\cdot}10) &= -\,0{\cdot}01 \\ 5{\cdot}1 - (-6{\cdot}17 + 19{\cdot}53 - 8{\cdot}23) &= -\,0{\cdot}03 \\ 2{\cdot}4 - (-2{\cdot}79 - 9{\cdot}35 + 14{\cdot}53) &= 0{\cdot}01 \end{aligned}$$

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DGEE/Promotion EO5 **Wednesday,September 16,2009**

Numerical Methods
Make up Exam

Ex1:

Numerically approximate the integral $\int_0^3 (3e^{-x} si(x^2)+1)dx$ by using the trapezoidal rule with n = 1, 2, 4, 8, and 16 subintervals.

Ex2:

Give the algorithm and program of Gauss elimination

Ex3:

Prove the statements:

$$\delta^3 f_j = f_{j+\frac{3}{2}} - 3f_{j+\frac{1}{2}} + 3f_{j-\frac{1}{2}} - f_{j-\frac{3}{2}}$$

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Numerical Methods
Make up Exam

Ex1:

Use the Newton-Raphson method to find to 4*S* the (positive) root of $3xe^x=1$?

Ex2:

Given that f (-2) = 46, f (-1) = 4, f (1) = 4, f (3) = 156, and f (4) = 484, use **Lagrange's interpolation formula** to estimate the value of f(0).

Ex3:

Find the inverse of the following matrix, using **elimination and back-substitution**

$$\begin{bmatrix} 2 & 6 & 4 \\ 6 & 19 & 12 \\ 2 & 8 & 14 \end{bmatrix}$$

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Make up (Solution)

Ex1:

Since $x > 0$, the root of $f(x) = \log_e 3x + x = 0$ must lie in the interval $0 < x < \frac{1}{3}$, where $\log_e 3x < 0$. If $x_0 = 0{\cdot}25$ is the initial guess,

$$f(0{\cdot}25) = \log_e(0{\cdot}75) + 0{\cdot}25 = -0{\cdot}2877 + 0{\cdot}25 = -0{\cdot}0377.$$

Since

$$f'(x) = \frac{1}{x} + 1,$$

$$f'(0{\cdot}25) = 5$$

and

$$x_1 = 0{\cdot}25 + \frac{0{\cdot}0377}{5} = 0{\cdot}25 + 0{\cdot}0075 = 0{\cdot}2575.$$

Then

$$f(0{\cdot}2575) = \log_e(0{\cdot}7725) + 0{\cdot}2575 = -0{\cdot}2581 + 0{\cdot}2575 = -0{\cdot}0006$$

$$f'(0{\cdot}2575) = 3{\cdot}883 + 1 = 4{\cdot}883,$$

and

$$x_2 = 0{\cdot}2575 + \frac{0{\cdot}0006}{4{\cdot}883} = 0{\cdot}2575 + 0{\cdot}0001 = 0{\cdot}2576.$$

Since $f(0{\cdot}2576) = -0{\cdot}0001$, we conclude that the root is $0{\cdot}2576$.

$$x_{n+1} = \tfrac{1}{3} e^{-x_n}$$

Ex2:

The Lagrange coefficients are

$$L_0(x) = \frac{(x+1)(x-1)(x-3)(x-4)}{(-1)(-3)(-5)(-6)} \quad \text{for } x_0 = -2,$$

$$L_1(x) = \frac{(x+2)(x-1)(x-3)(x-4)}{1(-2)(-4)(-5)} \quad \text{for } x_1 = -1,$$

$$L_2(x) = \frac{(x+2)(x+1)(x-3)(x-4)}{3\times 2(-2)(-3)} \quad \text{for } x_2 = 1,$$

$$L_3(x) = \frac{(x+2)(x+1)(x-1)(x-4)}{5\times 4\times 2(-1)} \quad \text{for } x_3 = 3,$$

$$L_4(x) = \frac{(x+2)(x+1)(x-1)(x-3)}{6\times 5\times 3\times 1} \quad \text{for } x_4 = 4.$$

Thus

$$f(0) = L_0(0)\times 46 + L_1(0)\times 4 + L_2(0)\times 4 + L_3(0)\times 156 + L_4(0)\times 484$$
$$= (-92 + 36 + 40 - 468 + 484)/15$$
$$= 0$$

Ex3:

m	A			I			*Check*	*Row operation*
	2	6	4	1	0	0	13	(1)
	6	19	12	0	1	0	38	(2)
	2	8	14	0	0	1	25	(3)
	2	6	4	1	0	0	13	(4) = (1)
−3	0	1	0	−3	1	0	−1	(5) = (2) − 3(1)
−1	0	2	10	−1	0	1	12	(6) = (3) − 1(1)
	2	6	4	1	0	0	13	(7) = (1)
	0	1	0	−3	1	0	−1	(8) = (5)
−2	0	0	10	5	−2	1	14	(9) = (6) − 2(5)
Inverse matrix				8·5	−2·6	−0·2		(Check that $AA^{-1} = I$)
				−3	1	0		
				0·5	−0·2	0·1		

Note: The first column of A^{-1} is $\begin{pmatrix} u_3 \\ u_2 \\ u_1 \end{pmatrix}$, found by solving

$$\begin{bmatrix} 2 & 6 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} u_3 \\ u_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 5 \end{bmatrix}$$ by back-substitution.

The second column is found by solving

$$\begin{bmatrix} 2 & 6 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} u_6 \\ u_5 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix};$$

the third is from

$$\begin{bmatrix} 2 & 6 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} u_9 \\ u_8 \\ u_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Ex1:

Given that f (-2) = 46, f (-1) = 4, f (1) = 4, f (3) = 156, and f (4) = 484, use **Lagrange's interpolation formula** to estimate the value of f(0).

Ex2:

Estimate by **numerical integration**.the value of the integral $\int_0^1 \frac{1}{1+x}dx$ with h=0.1 and bonded the error.

Sol 1:

The Lagrange coefficients are

$$L_0(x) = \frac{(x+1)(x-1)(x-3)(x-4)}{(-1)(-3)(-5)(-6)} \qquad \text{for } x_0 = -2,$$

$$L_1(x) = \frac{(x+2)(x-1)(x-3)(x-4)}{1(-2)(-4)(-5)} \qquad \text{for } x_1 = -1,$$

$$L_2(x) = \frac{(x+2)(x+1)(x-3)(x-4)}{3\times 2(-2)(-3)} \qquad \text{for } x_2 = 1,$$

$$L_3(x) = \frac{(x+2)(x+1)(x-1)(x-4)}{5\times 4\times 2(-1)} \qquad \text{for } x_3 = 3,$$

$$L_4(x) = \frac{(x+2)(x+1)(x-1)(x-3)}{6\times 5\times 3\times 1} \qquad \text{for } x_4 = 4.$$

Thus

$$\begin{aligned} f(0) &= L_0(0)\times 46 + L_1(0)\times 4 + L_2(0)\times 4 + L_3(0)\times 156 \\ &\qquad + L_4(0)\times 484 \\ &= (-92 + 36 + 40 - 468 + 484)/15 \\ &= 0 \end{aligned}$$

Sol 2:

We have

$$f(x) = \frac{1}{1+x}, \quad f''(x) = \frac{2}{(1+x)^3}, \quad f^{(4)}(x) = \frac{24}{(1+x)^5}.$$

x	0	0·1	0·2	0·3	0·4	0·5
$f(x)$	1·000000	0·909091	0·833333	0·769231	0·714286	0·666667
x	0·6	0·7	0·8	0·9	1·0	
$f(x)$	0·625000	0·588235	0·555556	0·526316	0·500000	

By Simpson's rule,

$$\begin{aligned}\int_0^1 \frac{1}{1+x}\,dx &= \frac{0{\cdot}1}{3}[1 + 4(0{\cdot}909091 + 0{\cdot}769231 + 0{\cdot}666667 \\ &\quad + 0{\cdot}588235 + 0{\cdot}526316) \\ &\quad + 2(0{\cdot}833333 + 0{\cdot}714286 + 0{\cdot}625000 \\ &\quad + 0{\cdot}555556) + 0{\cdot}500000] \\ &= 0{\cdot}6931(5).\end{aligned}$$

Ex3: (4p)

Prove the statements:

$$\delta^3 f_j = f_{j+\frac{3}{2}} - 3f_{j+\frac{1}{2}} + 3f_{j-\frac{1}{2}} - f_{j-\frac{3}{2}}$$

Sol 3

$$\begin{aligned}\delta^3 f_j &= \delta^2(\delta f_j) \\ &= \delta^2(f_{j+\frac{1}{2}} - f_{j-\frac{1}{2}}) \\ &= \delta(\delta f_{j+\frac{1}{2}} - \delta f_{j-\frac{1}{2}}) \\ &= \delta(f_{j+1} - 2f_j + f_{j-1}) \\ &= f_{j+\frac{3}{2}} - 3f_{j+\frac{1}{2}} - 3f_{j-\frac{1}{2}} - f_{j-\frac{3}{2}}.\end{aligned}$$

Ex4:

Estimate the value of the integral $\int_{1.0}^{1,3} \sqrt{x}\,dx$

Sol 4:

With $b - a = 1{\cdot}30 - 1{\cdot}00 = 0{\cdot}30$, we may choose

$$h = 0{\cdot}30, 0{\cdot}15, 0{\cdot}10, 0{\cdot}05, \ldots .$$

If $T(h)$ denotes the approximation corresponding to strip width h, we get

$$T(0{\cdot}30) = \frac{0{\cdot}30}{2}(1{\cdot}00000 + 1{\cdot}14018) = 0{\cdot}32102(7),$$

$$T(0{\cdot}15) = \frac{0{\cdot}15}{2}(1{\cdot}00000 + 1{\cdot}14018) + (0{\cdot}15)(1{\cdot}07238)$$

$$= 0{\cdot}16051(4) + 0{\cdot}16085(7) = 0{\cdot}32137(1),$$

$$T(0{\cdot}10) = \frac{0{\cdot}10}{2}(1{\cdot}00000 + 1{\cdot}14018) + (0{\cdot}10)(1{\cdot}04881 + 1{\cdot}09545)$$

$$= 0{\cdot}10700(9) + 0{\cdot}21442(6) = 0{\cdot}32143(5),$$

$$T(0{\cdot}05) = \frac{0{\cdot}05}{2}(1{\cdot}0000 + 1{\cdot}14018) +$$

$$+ (0{\cdot}05)(1{\cdot}02470 + 1{\cdot}04881 + 1{\cdot}07238 + 1{\cdot}09545 + 1{\cdot}11803)$$

$$= 0{\cdot}05350(5) + 0{\cdot}26796(9) = 0{\cdot}32147(4)$$

the answer is in fact $0{\cdot}32148537$, so the error sequence $0{\cdot}00045(8)$, $0{\cdot}00011(4)$, $0{\cdot}00005(0)$, $0{\cdot}00001(1)$

Ex5

Write the Pascal program:

1) For finding the mathematical root of the expression $Ax^2 + Bx + C$
2) To describe the weather according to the following temperature classifications:

greater than 75 hot
50 to 75 cool
35 to 49 cold
less than 35 freezing

Ex6:

Construct the **difference table** for the function $f(x) = x^3$ for x = 0(1) 6.

Sol 6:

x	$f(x)=x^3$	First difference	Second	Third	Fourth
1	1				
		7			
2	8		12		
		19		6	
3	27		18		0
		37		6	
4	64		24		0
		61		6	
5	125		30		
		91			
6	216				

Ex 7

Given the following function

x	1	2	3	4	5
F(x)	100.000	25.000	11.111	6.250	4.000

Extrapolate to find f(5.7) using one of the :

$$f\ (x) = f(0) + \frac{x}{h}\Delta f_0 + \frac{x(x-h)}{2!.h^2}\Delta^2 f_0 + \frac{x(x-h)(x-2h)}{3!.h^3}\Delta^3 f_0 + ...$$

$$f(x) = f(0) + \frac{x}{h}\nabla f_0 + \frac{x(x+h)}{2!.h^2}\nabla^2 f_0 + \frac{x(x+h)(x+2h)}{3!.h^3}\nabla^3 f_0 + ...$$

Ex8

Joe had decided to buy stocks of a particularly promising Internet company. The price per share was $100, and Joe subsequently recorded the stock price at the end of each week. With the abscissae measured in days, the following data were acquired:
(0; 100); (7; 98); (14; 101); (21; 50); (28; 51); (35; 50).
In attempting to analyze what happened, it was desired to approximately evaluate the stock price a few days before the crash.
(a) Pass a linear interpolant through the points with abscissae 7 and 14. Then add to this data set the value at 0 and (separately) the value at 21 to obtain two quadratic interpolants. Evaluate all three interpolants at x = 12. Which one do you think is the most accurate? Explain.
(b) Plot the two quadratic interpolants above, together with the data (without a broken line passing through the data) over the interval [0; 21]. What are your observations?

Ex9:

Give the program of polynomial method

Ex10:

Use Taylor series to find the truncation errors in the formulae:

- $f''(x_j) \approx \left[f(x_j+2h) - 2f(x_j+h) + f(x_j)\right]/h^2$

Sol 10

First we have:

Expanding about $x = x_j$:

$f(x_j + h) = f(x_j) + hf'(x_j) + \frac{1}{2}h^2 f''(x_j) + \ldots,$

so $(f(x_j + h) - f(x_j))/h = f'(x_j) + \frac{1}{2}hf''(x_j) + \ldots$

and the error $\approx \frac{1}{2}hf''(x_j)$.

Second we have:

Expanding about $x = x_j + \frac{1}{2}h$:

$$f(x_j + h) = f(x_j + \tfrac{1}{2}h) + \tfrac{1}{2}hf'(x_j + \tfrac{1}{2})$$
$$+ \tfrac{1}{8}h^2f''(x_j + \tfrac{1}{2}h) + \tfrac{1}{48}h^3f'''(x_j + \tfrac{1}{2}h) + \ldots$$

and $f(x_j) = f(x_j + \tfrac{1}{2}h) - \tfrac{1}{2}hf'(x_j + \tfrac{1}{2}h) + \tfrac{1}{8}h^2f''(x_j + \tfrac{1}{2}h)$
$$- \tfrac{1}{48}h^3f'''(x_j + \tfrac{1}{2}h) + \ldots$$

so $(f(x_j + h) - f(x_j))/h = f'(x_j + \tfrac{1}{2}h) + \tfrac{1}{24}h^2f'''(x_j + \tfrac{1}{2}h) + \ldots$

and the error $\approx \tfrac{1}{24}h^2f'''(x_j + \tfrac{1}{2}h)$.

Then:

Expanding about $x = x_j$:

$f(x_j + 2h) = f(x_j) + 2hf'(x_j) + 2h^2f'''(x_j) + \tfrac{4}{3}h^3f'''(x_j) + \ldots,$

so $(f(x_j + 2h) - 2f(x_j + h) + f(x_j))/h^2 = f''(x_j) + hf'''(x_j) + \ldots$

and the error $\approx hf'''(x_j)$.

Numerical Methods
Synthesis Exam

Ex1:

Use the **bisection method** to find the root of the equation : x+cosx = 0. correct to two decimal places (2D).

Ex2:

Solve the following system by Gauss elimination:

$$x_1 + x_2 - x_3 = 0,$$
$$2x_1 - x_2 + x_3 = 6,$$
$$3x_1 + 2x_2 - 4x_3 = -4.$$

Ex3:

Construct the **difference table** for the function $f(x) = x^3$ for x = 0(1) 6.

Ex4:

Find the inverse of the following matrix, using **elimination and back-substitution**

$$\begin{bmatrix} 2 & 6 & 4 \\ 6 & 19 & 12 \\ 2 & 8 & 14 \end{bmatrix}$$

Ex5:

Estimate the value of the integral $\int_{1.0}^{1,3} \sqrt{x}dx$

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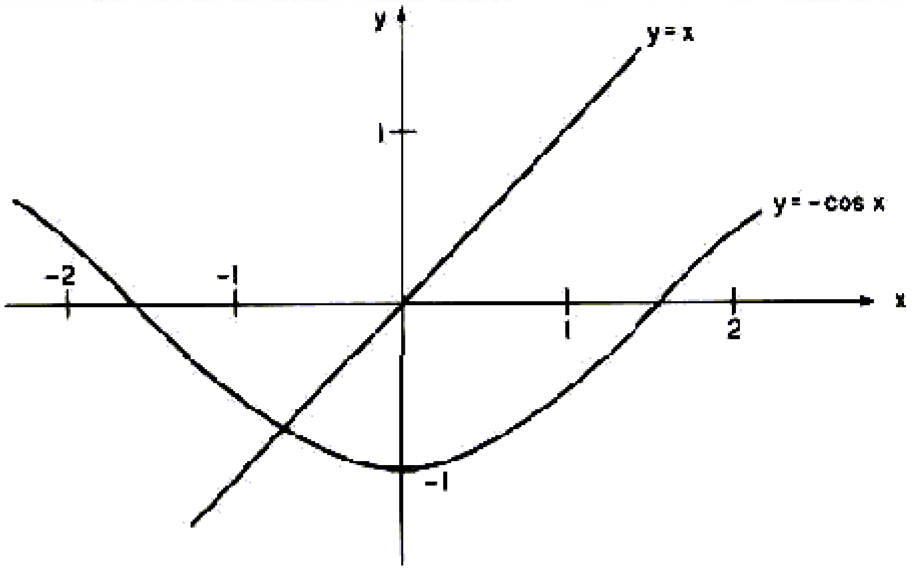
Numerical Methods
Synthesis Solution

Ex1:

The curves are sketched in Fig below one real root near $x = -0{\cdot}7$.
Tabulating confirms this:

x	$-0{\cdot}7$	$-0{\cdot}8$	$-0{\cdot}75$
$\cos x$	$0{\cdot}7648$	$0{\cdot}6967$	$0{\cdot}7317$
$x + \cos x$	$0{\cdot}0648$	$-0{\cdot}1033$	$-0{\cdot}0183$



Graphs of $y = x$ and $y = -\cos x$

Ex2:

m	Augmented Matrix				Check
	1	1	−1	0	1
	2	−1	1	6	8
	3	2	−4	−4	−3
	1	1	−1	0	1
−2		−3	3	6	6
−3		−1	−1	−4	−6
	1	1	−1	0	1
		−3	3	6	6
−1/3			−2	−6	−8

Solution by back-substitution *Residuals*

$$\left.\begin{array}{lcll} -2x_3 & = & -6 & \to x_3 = 3 \\ -3x_2 + 9 & = & 6 & \to x_2 = 1 \\ x_1 + 1 - 3 & = & 0 & \to x_1 = 2 \end{array}\right\} \quad \begin{array}{l} 0 - (2 + 1 - 3) = 0 \\ 6 - (4 - 1 + 3) = 0 \\ -4 - (6 + 2 - 12) = 0 \end{array}$$

Ex3:

x	$f(x)=x^3$	First difference	Second	Third	Fourth
1	1				
		7			
2	8		12		
		19		6	
3	27		18		0
		37		6	
4	64		24		0
		61		6	
5	125		30		
		91			
6	216				

Ex4:

m	**A**			**I**			*Check*	*Row operation*
	2	6	4	1	0	0	13	(1)
	6	19	12	0	1	0	38	(2)
	2	8	14	0	0	1	25	(3)
	2	6	4	1	0	0	13	(4) = (1)
−3	0	1	0	−3	1	0	−1	(5) = (2) − 3(1)
−1	0	2	10	−1	0	1	12	(6) = (3) − 1(1)
	2	6	4	1	0	0	13	(7) = (1)
	0	1	0	−3	1	0	−1	(8) = (5)
−2	0	0	10	5	−2	1	14	(9) = (6) − 2(5)
Inverse matrix				8·5	−2·6	−0·2		(Check that $AA^{-1} = I$)
				−3	1	0		
				0·5	−0·2	0·1		

Note: The first column of A^{-1} is $\begin{pmatrix} u_3 \\ u_2 \\ u_1 \end{pmatrix}$, found by solving

$$\begin{bmatrix} 2 & 6 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} u_3 \\ u_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 5 \end{bmatrix}$$ by back-substitution.

The second column is found by solving

$$\begin{bmatrix} 2 & 6 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} u_6 \\ u_5 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix};$$

the third is from

$$\begin{bmatrix} 2 & 6 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} u_9 \\ u_8 \\ u_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Ex5:

With $b-a = 1{\cdot}30 - 1{\cdot}00 = 0{\cdot}30$, we may choose

$$h = 0{\cdot}30, 0{\cdot}15, 0{\cdot}10, 0{\cdot}05, \ldots .$$

If $T(h)$ denotes the approximation corresponding to strip width h, we get

$$T(0{\cdot}30) = \frac{0{\cdot}30}{2}(1{\cdot}00000 + 1{\cdot}14018) = 0{\cdot}32102(7),$$

$$T(0{\cdot}15) = \frac{0{\cdot}15}{2}(1{\cdot}00000 + 1{\cdot}14018) + (0{\cdot}15)(1{\cdot}07238)$$

$$= 0{\cdot}16051(4) + 0{\cdot}16085(7) = 0{\cdot}32137(1),$$

$$T(0{\cdot}10) = \frac{0{\cdot}10}{2}(1{\cdot}00000 + 1{\cdot}14018) + (0{\cdot}10)(1{\cdot}04881 + 1{\cdot}09545)$$

$$= 0{\cdot}10700(9) + 0{\cdot}21442(6) = 0{\cdot}32143(5),$$

$$T(0{\cdot}05) = \frac{0{\cdot}05}{2}(1{\cdot}0000 + 1{\cdot}14018) +$$

$$+ (0{\cdot}05)(1{\cdot}02470 + 1{\cdot}04881 + 1{\cdot}07238 + 1{\cdot}09545 + 1{\cdot}11803)$$

$$= 0{\cdot}05350(5) + 0{\cdot}26796(9) = 0{\cdot}32147(4)$$

the answer is in fact 0·32148537, so the error sequence 0·00045(8), 0·00011(4), 0·00005(0), 0·00001(1)

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Numerical Methods
Synthesis Exam

Ex1

Given the following function

x	1	2	3	4	5
F(x)	100.000	25.000	11.111	6.250	4.000

Extrapolate to find f(5.7) using one of the :

$$f\ (x) = f(0) + \frac{x}{h}\Delta f_0 + \frac{x(x-h)}{2!.h^2}\Delta^2 f_0 + \frac{x(x-h)(x-2h)}{3!.h^3}\Delta^3 f_0 + ...$$

$$f(x) = f(0) + \frac{x}{h}\nabla f_0 + \frac{x(x+h)}{2!.h^2}\nabla^2 f_0 + \frac{x(x+h)(x+2h)}{3!.h^3}\nabla^3 f_0 + ...$$

Ex2

Give the algorithm and the program of Lagrange interpolation using Fortran or Pascal

Ex3

Joe had decided to buy stocks of a particularly promising Internet company. The price per share was $100, and Joe subsequently recorded the stock price at the end of each week. With the abscissae measured in days, the following data were acquired:
(0; 100); (7; 98); (14; 101); (21; 50); (28; 51); (35; 50).
In attempting to analyze what happened, it was desired to approximately evaluate the stock price a few days before the crash.
(a) Pass a linear interpolant through the points with abscissae 7 and 14. Then add to this data set the value at 0 and (separately) the value at 21 to obtain two quadratic interpolants. Evaluate all three interpolants at x = 12. Which one do you think is the most accurate? Explain.
(b) Plot the two quadratic interpolants above, together with the data (without a broken line passing through the data) over the interval [0; 21]. What are your observations?

Ex4

Give the algorithm and the program of Simpson integration using Fortran or Pascal

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Ex2

Algorithm: Lagrange polynomial interpolation.

1. *Construction*: Given data $\{(x_i, y_i)\}_{i=0}^{n}$, compute $\rho_j := \prod_{i \neq j}(x_j - x_i)$ for $j = 0, 1, \ldots n$.
2. *Evaluation*: Given an evaluation point x,
   (a) compute $\psi(x) = \prod_{i=0}^{n}(x - x_i)$;
   (b) compute $p(x) = \psi(x) \sum_{j=0}^{n} \frac{y_j}{(x - x_j)\rho_j}$.

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Numerical Methods
Synthesis Exam

Ex1

Using the finite differences, the following data represent a polynomial of what degree? What is the coefficient of the highest degree term?

x	0	1	2	3	4	4.5	5
F(x)	1	0.5	8	35.5	95.0	96.2	198.5

Ex2

Find $\sqrt{7}$ using numerical methods.

Ex3

Given the following data and using trapezoidal integration evaluate the integral:

$$I = \int_{0}^{1.2} f(x)dx$$

x	0	0.1	0.2	0.3	0.4	0.6	0.7	0.8	0.9	1.0	1.1	1.2
F(x)	93	87	68	55	42	35	39	48	53	51	39	37

Ex4

Give the flowchart of Gauss elimination.

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Ex1

Possibilities of answers :

1) Problem with step so we can't solve it with finite differences.
2) Problem with step so eliminate 4.5 and we use finite differences.
3) Problem with step so use Lagrange

Higher Degree =3

$F(x) = 2x^3 + C_1\frac{x^2}{2} + C_2x + C_3$

Ex2

$F(x) = x^2 - 7$ so $F'(x) = 2x$ and starting with x_0=3 ,working with Newton iterations

First iteration : x=2.6666

Second iteration : x=2.6458

Third iteration: x=2.64575

X= 2.64575

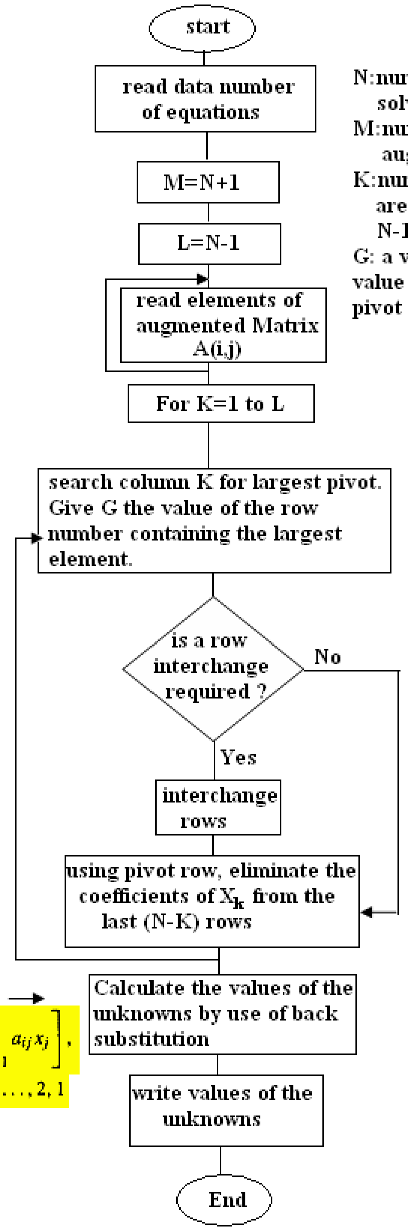
Ex3

1) Problem with step so we can't solve it with Trapezoidal integration.
2) Problem with step so we decompose into two sub integrals.

I= 54.35

Ex4

Flow chart of Gauss elimination method



N:number of equations in set be solved.
M:number of columns in the augmented matrix A(i,j).
K:number of rows in matrix which are possible pivot rows (equal to N-1).
G: a variable which takes on the value of the row having the largest pivot element.

in this step:

$m_{ik} = a_{ik}/a_{kk}, \quad i = k+1, k+2, \ldots, n$

$a_{ij} = a_{ij} - m_{ik}a_{kj}, \quad b_i = b_i - m_{ik}b_k.$

in this step:

$$x_i = \frac{1}{a_{ii}}\left[b_i - \sum_{j=i+1}^{n} a_{ij}x_j\right],$$

$i = n-1, n-2, \ldots, 2, 1$

Numerical Methods
Make up

Ex1

(Give the answer in this sheet)

```
program  CompleteThisProgram
type
………………………………………………….
…………………………………………………..
…………………………………………………..
………………………………………………….
………………………………………………….
…………………………………………………..
………………………………………………….
………………………………………………….
………………………………………………….
 begin
{    Data reading }
   writeln('Enter n');
   read(n);
          m:=n+1;
          l:=n-1;
{   Enter the augmented matrix } ;
   for i:=1 to n do
      for j:=1 to m do
         writeln('elements of   a');
          readln(a[i,j]);

{   Main  Work }
   for k:=1 to l do
      {  Rows Interchange }
       G:=k;
            for i:=k+1 to n do
              if (abs(a[i,k])>abs(a[G,k]))then
             ……………………………

             if (G<>k)then
               for j:=k to m  do
                  begin
                     ………………………………………….
                     ………………………………………….
                     ………………………………………..
                     ………………………………………….

   {     2nd step }
     for i:=k+1 to n do
         multi:=a[i,k]/a[k,k]
          for j:=k to m do
               a[i,j]:=a[i,j]-(multi*a[k,j]);
           end;
        end;

  {     3rd step  }
       for i:=n downto 1
       x[i]:=a[i,m];
        for j:=i+1 to n do
           …………………………….
          end
       x[i]:=x[i]/a[i,i]
     end
      end
```

Ex2 :

Using interpolation prove that:

$y_1 = y_3 - 0.3(y_5 - y_{-3}) + 0.2(y_{-3} - y_{-5})$ with y_x : are the values of y corresponding to the arguments x.

Ex3

Establish the iterative formula to calculate the cube root of N.

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Solution of numerical Methods

Ex1

1)
```
amatrix=array[0..9,0..9] of real;
bmatrix=array[0..9,0..9] of real;
multimatrix=array[0..9,0..9] of real;
var temp,x:real;
a:amatrix;
b:bmatrix;
multi:multimatrix;
i,j,n,m,l,k,G:integer;
```

2)
```
G:=i;
```

3)
```
temp:=a[k,j];
a[k,j]:=a[G,j];
a[G,j]:=temp;
end;
```

4)
```
x[i]:=x[i]-(a[i,j]*x[j])
```

Ex2 :

the..points.(x, y).are.$(-5, y_{-5}), (-3, y_{-3}), (3, y_3)$.and.$(5, y_5)$

$$y_x = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_{-5} + \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_{-3} +$$

$$\frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3 + \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_5$$

Taking : $x_1 = -5$, $x_2 = -3$, $x_3 = 3$, $x_4 = 5$ and $x = 1$ then

$$y_1 = -\frac{y_{-5}}{5} + \frac{y_{-3}}{2} + y_3 - \frac{3}{10} y_5$$

$$\mathbf{y_1 = y_3 - 0.3(y_5 - y_{-3}) + 0.2(y_{-3} - y_5)}$$

Ex3

We have: $x = \sqrt[3]{N} \Rightarrow x^3 - N = 0$

Take $f(x) = x^3 - N$ so that $f'(x) = 3x^2$

by Newton $x_{n+1} = x_n - \left[\frac{f(x_n)}{f'(x_n)}\right]$

then $x_{n+1} = x_n - \frac{x_n^3 - N}{3x_n^2}$ so $\mathbf{x_{n+1} = \frac{1}{3}\left[2x_n + \frac{N}{x_n^2}\right]}$

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Boumerdes university Sunday, 18 /02/ 2007

Faculty of Engineering Sciences

Numerical Methods Synthesis Exam

Ex1: (4p)

Estimate the value of the integral $\int_{1.0}^{1,3} \sqrt{x}dx$

Ex2: (6p)

Give the program of polynomial method

Ex3: (6p)

a) Prove the statements:

$$\delta^3 f_j = f_{j+\frac{3}{2}} - 3f_{j+\frac{1}{2}} + 3f_{j-\frac{1}{2}} - f_{j-\frac{3}{2}}$$

b) Use Taylor series to find the truncation errors in the formulae:

- $f''(x_j) \approx \left[f(x_j + 2h) - 2f(x_j + h) + f(x_j)\right] / h^2$

Ex4: (4p)

Give the program of trapezoidal integration

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Solution :

Ex1:

With $b-a = 1{\cdot}30 - 1{\cdot}00 = 0{\cdot}30$, we may choose

$$h = 0{\cdot}30, 0{\cdot}15, 0{\cdot}10, 0{\cdot}05, \ldots .$$

If $T(h)$ denotes the approximation corresponding to strip width h, we get

$$T(0{\cdot}30) = \frac{0{\cdot}30}{2}(1{\cdot}00000 + 1{\cdot}14018) = 0{\cdot}32102(7),$$

$$T(0{\cdot}15) = \frac{0{\cdot}15}{2}(1{\cdot}00000 + 1{\cdot}14018) + (0{\cdot}15)(1{\cdot}07238)$$

$$= 0{\cdot}16051(4) + 0{\cdot}16085(7) = 0{\cdot}32137(1),$$

$$T(0{\cdot}10) = \frac{0{\cdot}10}{2}(1{\cdot}00000 + 1{\cdot}14018) + (0{\cdot}10)(1{\cdot}04881 + 1{\cdot}09545)$$

$$= 0{\cdot}10700(9) + 0{\cdot}21442(6) = 0{\cdot}32143(5),$$

$$T(0{\cdot}05) = \frac{0{\cdot}05}{2}(1{\cdot}0000 + 1{\cdot}14018) +$$

$$+ (0{\cdot}05)(1{\cdot}02470 + 1{\cdot}04881 + 1{\cdot}07238 + 1{\cdot}09545 + 1{\cdot}11803)$$

$$= 0{\cdot}05350(5) + 0{\cdot}26796(9) = 0{\cdot}32147(4)$$

the answer is in fact $0{\cdot}32148537$, so the error sequence $0{\cdot}00045(8)$, $0{\cdot}00011(4)$, $0{\cdot}00005(0)$, $0{\cdot}00001(1)$

Ex2:

Donne on Lab

Ex3:

a)

$$\begin{aligned}
\delta^3 f_j &= \delta^2(\delta f_j) \\
&= \delta^2(f_{j+\frac{1}{2}} - f_{j-\frac{1}{2}}) \\
&= \delta(\delta f_{j+\frac{1}{2}} - \delta f_{j-\frac{1}{2}}) \\
&= \delta(f_{j+1} - 2f_j + f_{j-1}) \\
&= f_{j+\frac{3}{2}} - 3f_{j+\frac{1}{2}} - 3f_{j-\frac{1}{2}} - f_{j-\frac{3}{2}}.
\end{aligned}$$

b)

First we have:

Expanding about $x = x_j$:

$f(x_j + h) = f(x_j) + hf'(x_j) + \frac{1}{2}h^2 f''(x_j) + \ldots,$

so $(f(x_j + h) - f(x_j))/h = f'(x_j) + \frac{1}{2}hf''(x_j) + \ldots$

and the error $\approx \frac{1}{2}hf''(x_j)$.

Second we have:

Expanding about $x = x_j + \frac{1}{2}h$:

$$f(x_j + h) = f(x_j + \tfrac{1}{2}h) + \tfrac{1}{2}hf'(x_j + \tfrac{1}{2}) + \tfrac{1}{8}h^2 f''(x_j + \tfrac{1}{2}h) + \tfrac{1}{48}h^3 f'''(x_j + \tfrac{1}{2}h) + \ldots$$

and $f(x_j) = f(x_j + \frac{1}{2}h) - \frac{1}{2}hf'(x_j + \frac{1}{2}h) + \frac{1}{8}h^2 f''(x_j + \frac{1}{2}h) - \frac{1}{48}h^3 f'''(x_j + \frac{1}{2}h) + \ldots$

so $(f(x_j + h) - f(x_j))/h = f'(x_j + \frac{1}{2}h) + \frac{1}{24}h^2 f'''(x_j + \frac{1}{2}h) + \ldots$

and the error $\approx \frac{1}{24}h^2 f'''(x_j + \frac{1}{2}h)$.

Then:

Expanding about $x = x_j$:

$f(x_j + 2h) = f(x_j) + 2hf'(x_j) + 2h^2 f'''(x_j) + \frac{4}{3}h^3 f'''(x_j) + \ldots,$

so $(f(x_j + 2h) - 2f(x_j + h) + f(x_j))/h^2 = f''(x_j) + hf'''(x_j) + \ldots$

and the error $\approx hf'''(x_j)$.

Ex4:

Donne on Lab

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Numerical Methods
Synthesis Exam

Ex1 (10p)

Solve this initial value problem numerically on the interval $0 \leq x \leq 2$, using Euler's method and Runge-Kutta method (two and four order). Use a constant step size of h=0.05.

$$y' = 2xy + 1, \qquad y(0) = 2$$

Put the results on the table.

X	Y^E	Y^{RK}

Which result do you prefer and why?

Ex2: (5p)

Consider the system defined by a group of inputs/outputs: $X^i = 1,2,3$ and $Y^i = 2,1,2$

Estimate the output corresponding to 1.5

Ex3: (5p)

Compute the integral using Simpson $I = \int f(x)dx = \int_0^{(\pi)^{1/2}} x.\sin(x^2)dx$ with four fixed points.

Compare this result to the exact one. How do you justify this?

$$k_1 = hf(x_n, y_n)$$
$$k_2 = hf(x_n + h/2, y_n + k_1/2)$$
$$k_3 = hf(x_n + h/2, y_n + k_2/2)$$
$$k_4 = hf(x_n + h, y_n + k_3)$$
$$y_{n+1} = y_n + \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_n, y_n)$$
$$k_2 = hf(x_n + h, y_n + k_1)$$
$$y_{n+1} = y_n + \tfrac{1}{2}(k_1 + k_2)$$

$$y_{n+1} = y_n + hf(x_n, y_n)$$

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Solution

Ex1:(10p)

Here, Y^E denotes the Euler's method estimates of y(t) , Y^{RK} denotes the Runge-Kutta method estimates.

X	Y^E	Y^{RK}(fourth)
0	2.0000	2.0000
0.05 **(0.5p)**	2.0500 **(0.5p)**	2.0551 **(0.5p)**
0.10 **(0.5p)**	2.1102 **(0.5p)**	2.1207 **(0.5p)**
0.15 **(0.5p)**	2.1813 **(0.5p)**	2.1978 **(0.5p)**
0.20 **(0.5p)**	2.2640 **(0.5p)**	2.2870 **(0.5p)**

The best result is given by Runge-Kutta method **(2p)**
Because in **Euler's method** the **truncation error** is large and the results are inaccurate (we obtain the formula of Euler from Taylor series with truncation of third term and up), the Runge-Kutta is obviously superior. **(2p)**

Ex2: (5p)

Using Lagrange formula, the polynomials are:

$L_1 = \frac{1}{2}(x^2 - 5x + 6)$ **(1p)** , $L_2 = -(x^2 - 4x + 3)$ **(1p)** , $L_1 = \frac{1}{2}(x^2 - 3x + 2)$ **(1p)**
So $P(x) = x^2 - 4x + 5$ **(1p)**
The output corresponding to 1.5 is $P(1.5) = 1.65$ **(1p)**

Ex 3) (5p)

The integral $I = \int f(x)dx = \int_0^{(\pi)^{1/2}} x.sin(x^2)dx$

$$S(n) = \frac{b-a}{3n}\left[f(x_0) + 4 f(x_1) + 2 f(x_2) + 4 f(x_3) + \ldots + 4 f(x_{n-1}) + f(x_n)\right]$$

1st)To apply Simpson's rule "n" must be even so we take n=4 $\Leftrightarrow f_0, f_1, f_2, f_3$ and f_4, then $h = \frac{b-a}{n} = \frac{(\pi)^{1/2}}{4}$
(1p)

x	0	$\frac{(\pi)^{1/2}}{4}$	$\frac{2(\pi)^{1/2}}{4}$	$\frac{3(\pi)^{1/2}}{4}$	$(\pi)^{1/2}$
f(x)	0	0.00151	0.0121	0.04099	0.0971

$$S = \frac{(\pi)^{1/2}}{12}[0 + 4 * 0.00151 + 2 * 0.0121 + 4 * 0.04099 + 0.0971] = 0.043 \quad \textbf{(1p)}$$

$$I = \int_0^{(\pi)^{1/2}} x.sin(x^2)dx = -\frac{1}{2}\cos(x^2)\Big]_0^{(\pi)^{1/2}} = 1 \quad \textbf{(1p)}$$

Calculating the error: $E = I - S = 0.9569$. We conclude a big difference and to avoid this problem we must take a great number of subintervals (great number of data) or we take a small length "h". (2p)

2nd)If we take n=2 $\Leftrightarrow f_0, f_1$ and f_2 , then $h = \frac{b-a}{n} = \frac{(\pi)^{1/2}}{2}$

x	0	$\frac{(\pi)^{1/2}}{2}$	$(\pi)^{1/2}$
f(x)	0	0.0121	0.0971

$$S = \frac{(\pi)^{1/2}}{6}[0 + 4 * 0.0121 + 0.0971] = 0.0535$$

$$E = I - S = 0.9465$$

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Numerical Methods Synthesis

Ex1

Use the Newton-Raphson formula to obtain a better estimate of the root of: $f(x) = x-2+\ln x$

Ex2

Give the approximation to f(1.5)of the first and second degree obtained by the various list.

x	F(x)
1.0	0.7651977
1.3	0.6200860
1.6	0.4554022
1.9	0.2818186
2.2	0.1103623

Ex3

Let us use Gauss Elimination Method to solve the following system:

$$\begin{cases} x - y + 2z = 5 \\ 3x + 2y + z = 10 \\ 2x - 3y - 2z = -10 \end{cases}$$

Ex4

Give the Lagrange polynomial of degree n for the function $f(x) = \cos(x)$ over the interval [0.0, 1.2] using equally spaced length.

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Solution:

Ex1:(5 points)

Here $x_0 = 1.5$, $f(1.5) = -0.5 + \ln(1.5) = -0.0945$ [
$f'(x) = 1 + \frac{1}{x}$ $\therefore$ $f'(1.5) = 1 + \frac{1}{1.5} = \frac{5}{3}$
Hence using the formula:

$$x_1 = 1.5 - \frac{(-0.0945)}{(1.6667)} = 1.5567$$

The Newton-Raphson formula can be used again: this time beginning with 1.5567 as our initial estimate.

This time use:

$$\begin{aligned} x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\ &= 1.5567 - \frac{f(1.5567)}{f'(1.5567)} \\ &= 1.5567 - \frac{\{1.5567 - 2 + \ln(1.5567)\}}{\{1 + \frac{1}{1.5567}\}} \\ &= 1.5567 - \frac{\{-0.0007\}}{\{1.6424\}} = 1.5571 \end{aligned}$$

Ex2: (5 points)

A)

Since 1.5 is between 1.3 and 1.6, the most appropriate linear polynomial uses $x_0 = 1.3$ and $x_1 = 1.6$. The value of the interpolating polynomial at 1.5 is

$$P_1(1.5) = \frac{(1.5-1.6)}{(1.3-1.6)}(0.6200860) + \frac{(1.5-1.3)}{(1.6-1.3)}(0.4554022) = 0.5102968.$$

B)

Two polynomials of degree 2 can reasonably be used, one by letting $x_0 = 1.3$, $x_1 = 1.6$, and $x_2 = 1.9$, which gives

$$\begin{aligned} P_2(1.5) = &\frac{(1.5-1.6)(1.5-1.9)}{(1.3-1.6)(1.3-1.9)}(0.6200860) + \frac{(1.5-1.3)(1.5-1.9)}{(1.6-1.3)(1.6-1.9)}(0.4554022) \\ &+ \frac{(1.5-1.3)(1.5-1.6)}{(1.9-1.3)(1.9-1.6)}(0.2818186) \\ = &\ 0.5112857, \end{aligned}$$

and the other by letting $x_0 = 1.0$, $x_1 = 1.3$, and $x_2 = 1.6$, which gives

$$\hat{P}_2(1.5) = 0.5124715.$$

Ex 3): (5 points)

$$\left(\begin{array}{ccc|c} (1) & -1 & 2 & 5 \\ 3 & 2 & 1 & 10 \\ 2 & -3 & -2 & -10 \end{array}\right) \left(\begin{array}{ccc|c} 1 & -1 & 2 & 5 \\ 0 & (5) & -5 & -5 \\ 0 & -1 & -6 & -20 \end{array}\right) \left(\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & (-7) & -21 \end{array}\right)$$

From which we compute z = 3 and then, by back substitution, the remaining unknowns y = 2 and x = 1

Ex4: (5 points)

The polynomial that interpolates all the data is:
F(x) = 1.388889(-1.2+x)(-0.6+x) – 2.2926(-1.2+x)(0.0+x) + 0.503275(-0.6+x)(0.0+x)

Mrs BOUSHAKI

<u>Boumerdes university</u> Tuesday, 30 January 2007

<u>Faculty of Engineering Sciences</u>

<u>Numerical Methods Test of Lab</u>

<u>Ex1:</u> (6p)

Write the program of polynomial methods

<u>Ex2:</u> (6p)

Write the program to finding the $\sqrt{8}$ using Newton first order

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

<u>Ex3:</u> (8p)

In each of the following questions, evaluate the given code fragments. Investigate each of the fragments for the various starting values given on the right. Use Matlab to check your answers

1.
```
if n > 1
        m = n + 2
        else
        m = N - 2
  end
```
a) n = 7 m = ?
b) n = 0 m = ?
c) n = -7 m = ?

2.
```
if s <= 1
        t = 2z
  elseif s =10
        t = 9 – z
  elseif s < 100
        t = Sqrt(s)
  else
        t = s
  end
```
a) s = 1 t = ?
b) s = 7 t = ?
c) s = 57 t = ?
d) s = 300 t = ?

3.
```
if t >= 24
        z = 3*t + 1
        elseif t < 9
        z = t^2/3 - 2t
  else
        z = Sin(t)
  end
```
a) t = 50 h = ?
b) t = 19 h = ?
c) t = -6 h = ?
d) t = 0 h = ?

4.
```
if 0 < x < 7
        y = 4x
  elseif 7 < x < 55
        y = -10x
  else
        y = = 333
  end
```
a) x = -1 y = ?
b) x = 5 y = ?
c) x = 30 y = ?
d) x = 56 y = ?

–

<u>Mrs BOUSHAKI R.</u>