EE 479: Make-up Solutions

① $\begin{cases} f(\lambda) = \lambda^{3/2} \\ f'(\lambda) = \frac{3}{2}\sqrt{\lambda} \end{cases}$; $\Delta(\lambda) = \lambda^2 - 2\lambda + 1 = (\lambda-1)^2 \Rightarrow \lambda = 1 \; ; \; m = 2$

Since $f(1)$ exist and $f'(1)$ exist $\Rightarrow f(A)$ exist $\Rightarrow A^{3/2}$

$p(\lambda) \triangleq \alpha_0 + \alpha_1 \lambda \Rightarrow \begin{cases} \alpha_0 + \alpha_1 = 1 \\ \alpha_1 = 3/2 \end{cases} \Rightarrow \begin{cases} \alpha_0 = -1/2 \\ \alpha_1 = 3/2 \end{cases} \Rightarrow A^{3/2} = -\frac{1}{2} I + \frac{3}{2} A$

$A = \begin{pmatrix} -1/2 & 3/2 \\ -3/2 & 5/2 \end{pmatrix}$ //

② Λ is a 5×5 matrix with 2 main blocks $\begin{cases} \nearrow 3\times3 \text{ for } \lambda_1 = 1 \\ \searrow 2\times2 \text{ for } \lambda_2 = 2 \end{cases}$

Since $m(\lambda) = (\lambda-1)^2(\lambda-2)^2 \Rightarrow \begin{cases} \text{The largest Jordan block for } \lambda_1 = 1 \text{ is } 2 \\ \text{The largest Jordan block for } \lambda_2 = 2 \text{ is } 2 \end{cases}$

Hence $\Lambda = \left(\begin{array}{cc|c|cc} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{array}\right)$

③ $q_1 = \frac{\begin{pmatrix}1\\1\end{pmatrix}}{\sqrt{2}} = \frac{\sqrt{2}}{2}\begin{pmatrix}1\\1\end{pmatrix}$; $A'_2 = A_2 - (q_1^T A_2) q_1 = \begin{pmatrix}1\\-1\end{pmatrix} - \frac{\sqrt{2}}{2}\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix} \Rightarrow$

$q_2 = \frac{\sqrt{2}}{2}\begin{pmatrix}1\\-1\end{pmatrix} \Rightarrow Q = [q_1 \; q_2] = \begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ \sqrt{2}/2 & -\sqrt{2}/2 \end{pmatrix}$ and $R = \begin{pmatrix} q_1^T A_1 & q_1^T A_2 \\ 0 & q_2^T A_2 \end{pmatrix} = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$.

Hence $A = \underbrace{\frac{\sqrt{2}}{2}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}}_{Q}\underbrace{\begin{pmatrix}\sqrt{2} & 0\\0 & \sqrt{2}\end{pmatrix}}_{R}$

④ $x_1 = Px$ where P is the projection matrix on $R(A) \Rightarrow P = A(A^TA)^{-1}A^T = \begin{pmatrix} 5/6 & -1/3 & -1/6 \\ -1/3 & 1/3 & -1/3 \\ -1/6 & -1/3 & 5/6 \end{pmatrix}$

Hence $x_1 = Px = \begin{pmatrix}1/3\\-1/3\\1/3\end{pmatrix} \Rightarrow x_2 = x - x_1 = \begin{pmatrix}2/3\\4/3\\2/3\end{pmatrix}$.

⑤ a) $S = \left\{\alpha\begin{pmatrix}1\\1\\2\end{pmatrix} + \beta\begin{pmatrix}1\\-1\\0\end{pmatrix}\right\} = \left\{\begin{pmatrix}\alpha+\beta\\\alpha-\beta\\2\alpha\end{pmatrix}\right\}$. dim $S = 2$ since degree of freedom is 2

Hence S is a plane, with equation $z = x + y$ //

b) $x \in S^\perp \Rightarrow \begin{cases} (x \; y \; z)\begin{pmatrix}1\\1\\2\end{pmatrix} = 0 \\ (x \; y \; z)\begin{pmatrix}1\\-1\\0\end{pmatrix} = 0 \end{cases} \Rightarrow x = y = -z \Rightarrow$

$S^\perp = \left[\begin{pmatrix}1\\1\\-1\end{pmatrix}\right] \Rightarrow$ A basis for $S^\perp$ is $\left\{\begin{pmatrix}1\\1\\-1\end{pmatrix}\right\}$.